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To the Graduate Council:

I am submitting herewith a thesis written by Jarred Wade Cowart entitled "Design Considerations for Reinforced Concrete Masonry Walls Regarding Moment Magnification." I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Civil Engineering.

Richard Bennett, Major Professor

We have read this thesis and recommend its acceptance:

Harold Deatherage, Edwin Burdette

Accepted for the Council:

Dixie L. Thompson

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

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Accepted for the Council:

Linda R. Painter
Interim Dean of Graduate Studies

(Original signatures are on file with official student records.)

Design Considerations for Reinforced Concrete Masonry Walls
Regarding Moment Magnification

A Thesis Presented for
The Master of Science
Degree
The University of Tennessee, Knoxville

Jarred Wade Cowart
May 2007

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Dedication

This thesis is dedicated to my wife, Jami Cowart, whose patience allowed for the completion of my education, to my parents, Greg Cowart and Barbara Cowart, who sacrificed so I could get the education of my choice, to my brother, Jason Cowart, who introduced me to my current path, and to my son, Jack Cowart, who I hope will be inspired by everything I accomplish.

Abstract

Currently, regardless of the height of a reinforced masonry wall, engineers are required to consider moment magnification when using strength design provisions. An attempt at finding reasonable h/r limits for consideration of moment magnification is explored. Moment-curvature diagrams were developed using a spreadsheet to define the behavior of 8 inch and 12 inch concrete masonry walls. A series of models were created with axial load, out-of-plane load, height, reinforcing, and wall width as variables. The models were loaded out-of-plane in a finite element program until failure. The results were organized to produce the moment magnification of each wall. Results were summarized to determine height to radius of gyration (h/r) ratio limits for consideration of moment magnification.

For walls with axial loads over 5% of the compressive strength times the cross sectional area, practical limits were not found. The limits corresponded to short walls that would not aid engineers in design. For those walls with axial load of 5% and less, walls with h/r ratios of 24 or less would not have to consider the effects of moment magnification, walls with h/r ratios between 24 and 40 would use a simplified method of increasing the static moment by 10%, and walls with h/r ratios over 40 would have to consider current moment magnification procedures.

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Nomenclature

A_n	net cross sectional area
A_s	cross sectional area of tension steel
b	width of flange (in)
C_m	factor relating actual moment diagram to an equivalent uniform moment diagram
CCS	corresponding compression strain
CMU	concrete masonry unit
d	depth (in)
E_m	modulus of elasticity of masonry (psi)
E_s	modulus of elasticity of steel (psi)
e_u	eccentricity of P_{uf} (in)
f'_m	compressive strength of masonry
h	height of wall
kd	distance from extreme compression fiber to neutral axis (in)
M_{cr}	nominal cracking moment strength (lb-in)
M_{ser}	service moment at midheight of a member (lb-in)
M_p	factored primary moment at the section due to the end factored moments and lateral loads (lb-in)
M_u	factored moment (lb-in)
$MSJC$	Masonry Standards Joint Committee
P	axial load (lb)
P_{cr}	critical buckling axial load (lb)
P_f	applied factored axial load (lb)
P_{uf}	factored load from tributary floor or roof areas (lb)
r	radius of gyration (in)
t_f	thickness of flange (in)
t_w	thickness of web
w	out-of-plane load (lb/in)
w_u	out-of-plane factored uniformly distributed load (lb/in)
x	distance from neutral axis to point of maximum allowable compressive stress (in)
δ_u	deflection due to factored loads (in)
ϵ_m	strain in the extreme fiber of compression masonry
ϵ_s	strain in the tension steel

1. Introduction

Design codes are developed using the most accurate and up to date research available. In order to predict the behavior of structures, codes often use very complex and confusing equations and methods. Conversely, engineers are continuously asked to produce more and more complex designs with tighter deadlines. In order to meet client demands, engineers are faced with a choice: design based on all available information and spend more time doing so, or cut time and thusly design costs and produce a conservatively designed structure and pass the cost on to the construction phase. One way code committees can ease the burden of the engineer is to provide simplified methods for complex phenomena. These methods exist for wind and seismic loads and are inherently provided in many other facets of structural engineering codes.

One element of structures that often is neglected for the sake of saving design time is moment magnification. Theoretically, moment magnification should be utilized in the design of every element that experiences out-of-plane load in combination with axial load. Prime examples of these elements are exterior concrete or masonry walls. These walls undergo out-of-plane bending due to wind while experiencing axial load from floors above or from the building's roof system. Moment magnification occurs in an element when the out-of-plane and axial loads are applied simultaneously. If the axial load remains constant, the deflection of the element due to the out-of-plane load causes a small amount of eccentricity with respect to the axial load. This eccentricity induces an additional moment on the element, which then causes a deflection in addition to the original out-of-plane deflection. In essence, moment magnification is best described as

an increase in moment due to an increase in deflection with no increase in out-of-plane load.

The current masonry code, ACI 530-05 includes both strength design and allowable stress design provisions. For this analysis only strength design will be considered. In the current masonry code the factored moment is found with the following equation:

$$M_u = \frac{w_u h^2}{8} + P_{uf} \frac{e_u}{2} + P_u \delta_u \quad \text{Eq. 3-24}$$

Where M_u is the factored moment, w_u is the out-of-plane factored uniformly distributed load, h is the effective height of the wall, P_{uf} is the factored load from tributary floor or roof areas, e_u is the eccentricity of P_{uf} , P_u is the factored axial load, and δ_u is the deflection due to factored loads. The first term of the equation is simply the factored moment of a simple beam loaded uniformly. The second term is the moment due to the initial eccentricity of the applied factored axial load due to tributary floor or roof areas. The third term is a moment that is determined by multiplying the applied axial load times the deflection of the wall in the out-of-plane direction due to the uniform load. This third term is where the masonry code factors in moment magnification (Masonry Standards Joint Committee, 2004).

The Canadian masonry code, S304.1-94, considers moment magnification in a different way. Walls are divided into two categories, those with kh/t ratios less than or equal to 30, and those with ratios greater than 30. When the slenderness of the wall is greater than 30, the Canadian code calculates the moment in the same way as ACI 530-05 as shown above. MacGregor et al (1970) and MacGregor (1993) developed the basis for

the design of slender concrete columns that follows. For walls with kh/t ratios less than 30, the moment is calculated directly using the following equation:

$$M_{tot} = M_p \frac{C_m}{\left(1 - \frac{P_f}{P_{cr}}\right)} \quad \text{Sect. 11.2.5.3}$$

In this equation, M_p is the “factored primary moment at the section due to the end factored moments and lateral loads,” C_m is the moment diagram factor, P_{cr} is the critical buckling axial load, and P_f is the applied factored axial load (Canadian Standards Association, 1994). The rest of the equation is very similar to the ACI-318 method of determining moment for members in nonsway frames. The ACI 318-05 equation for moment including moment magnification is:

$$M_c = M_2 \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \geq 1.0 \quad \text{Eq. 10-9}$$

where M_2 is the larger factored end moment, C_m is the factor relating actual moment diagram to an equivalent uniform moment diagram, P_u is the factored axial force, and P_c is the critical buckling load. ACI 318-05 and the Canadian code use the same methodology, but have different factors embedded in the calculations. These equations multiply the first-order moment by a factor to take into account moment magnification (ACI Committee 318, 2004).

The American and Canadian masonry codes limit the deflection of walls. The Masonry Standards Joint Committee (MSJC) limits the deflection under service loads to:

$$\delta_s = 0.007h \quad \text{Eq. 3-29}$$

The Canadian code provides the following limitations:

- 1) where masonry veneers are attached – span/720
- 2) where brittle finishes are attached – span/360
- 3) otherwise – span/180

The deflection calculations are the same in both codes:

- 1) when the moment due to service loads is less than the cracked moment:

$$\delta_s = \frac{5M_{ser}h^2}{48E_m I_g} \quad \text{Eq. 3-30}$$

- 2) when the moment due to service loads is greater than the cracked moment and less than the ultimate moment

$$\delta_s = \frac{5M_{cr}h^2}{48E_m I_g} + \frac{5(M_{ser} - M_{cr})h^2}{48E_m I_{cr}} \quad \text{Eq. 3-31}$$

where M_{cr} is the nominal cracking moment strength, E_m is the modulus of elasticity of masonry in compression, I_g is the moment of inertia of gross cross-sectional area of a member, and M_{ser} is the service moment at midheight of a member (including P-delta effects). The basis for most of the code provisions dealing with deflection is the work of Horton and Tadros (1990).

Currently, the MSJC code does not consider moment magnification effects for unreinforced masonry. The draft 2008 code will include a moment magnifier approach for unreinforced masonry (Bennett, 2006). As part of the approach, limits are provided for which moment magnification effects do not need to be included. For walls with an h/r (height to radius of gyration) ratio of 40 or less, moment magnification can be neglected. The effects of moment magnification contribute less than 5% of the total moment when

the slenderness ratio is less than 40. For walls with h/r ratios between 40 and 60, either a moment magnifier should be used or the nominal strength shall be reduced by 10%. In this range, the maximum moment magnification is 10%, so requiring all walls in this range to increase the static moment by 10% would be conservative in most cases. For wall with h/r ratios over 60, a moment magnifier like that used in the American and Canadian concrete codes would have to be used. These criteria are used when looking at reinforced masonry walls for this analysis. Ideally, when the results of the models are analyzed, an h/r ratio for which the moment magnification contributes very little (5% or less) to the overall moment in the wall will be found and the moment magnification can therefore be neglected; an intermediate range of h/r ratios where a small reduction in strength will encompass the true increase in moment that moment magnification would contribute; and all other h/r ratios would have to consider moment magnification.

The methodology for this analysis is as follows: 1) develop moment vs. curvature diagrams for each cross section of wall to be analyzed, 2) develop a finite element model for each cross section for a variety of heights in order to get a wide range of h/r ratios, 3) compare the moment from the model to the static moment to obtain the moment magnification, and 4) analyze the results to obtain limits for the methodology of moment magnification calculations in reinforced masonry walls.

2. Development of Moment vs. Curvature Diagrams

Moment curvature diagrams will be used to model the non-linear behavior of reinforced masonry walls as cracking occurs. For this analysis, moment curvature diagrams for masonry bearing walls with varying axial load have to be established. In order to create these curves, a clear cross-section needs to be defined. The cross-section of a typical concrete masonry unit (CMU) wall can be seen in Figure 1 of the Appendix. The top and bottom flanges are considered to be as wide as the bar spacing and as thick as the face shell. This portion of the cross section is where the mortar is placed between courses. The web of the section consists of the grout used to confine the reinforcing steel as well as the adjoining sections of the masonry unit. The idealized cross-section is the basis for determining the moment-curvature diagrams.

A moment-curvature diagram provides an accurate means of defining the strength of a member in bending. In essence, this diagram shows the relationship between the moment the member is subjected to versus the interior angle of the stress diagram. Figure 2 illustrates curvature (Φ) as it relates to the strain diagram. For walls with typical amounts of steel reinforcing, the curvature increases as the moment increases. While a member is in the elastic range, the moment and curvature are proportional. After the member reaches the cracking moment and the curvature continues to increase, the moment will initially decrease. This decrease is attributed to the reduced cross section of the member. The cross-section is assumed to have cracked from the extreme tension fiber to the steel reinforcing. Theoretically, this portion of the member can no longer resist the

tension force due to bending, which will then be resisted by the reinforcing steel. After the small decrease, the moment will once again increase with continued bending. This increase in moment and curvature will continue non-linearly as the member continues to bend up to the yield moment (the point at which the steel reaches its yield stress.) After the yield moment has been reached and the member continues to bend, the increase in moment will continue to the ultimate moment. There is very little increase in moment from yield to ultimate, but the increase in curvature is typically very large. This is due to the stress-strain curve of reinforcing steel having a “flat top” once the yield stress has been reached.

Walls with very large amounts of reinforcing steel may not have moment-curvature diagrams that reach the yield point. The ultimate moment as defined by ACI-530 may be reached before the steel in the wall can yield. Conversely, walls with very little reinforcing steel may have yield moments and/or ultimate moments that are less than the cracking moment. If this happens, the wall will fail once the tension masonry cracks.

The variables that affect moment-curvature diagrams are masonry type, amount of steel, steel spacing, axial load, mortar type, and wall width. A spreadsheet was created to produce diagrams for walls with combinations of each variable. Table 1 lists each variable and its possible values. The number of moment-curvature diagrams available is very large. In order to encompass this wide range, a select number of diagrams were chosen. The diagrams used in the modeling process consist of combinations shown in Table 2. An effort was made to choose the most common physical parameters. In all, 40 unique moment-curvature diagrams were produced for analysis. ACI 530-05 provides a

maximum amount of reinforcing by requiring the strain in the tension steel at ultimate to be 1.5 times the yield strain. Twelve of the moment-curvature diagrams contain reinforcing that exceeds the maximum allowed by ACI 530-05. The effect of large amounts of reinforcing on moment magnification is of interest in this investigation; therefore, the walls violating this limit are included.

For simplicity, most diagrams were idealized as three lines connecting four points. These four points are: 1) zero moment, 2) cracking moment, 3) yield moment, and 4) ultimate moment along with their respective curvatures. Figure 3 shows an example of an idealized moment-curvature diagram along with a diagram showing the true shape of the moment-curvature relationship. The portion of the diagram between zero moment and cracking, and between yield and ultimate is the same for both diagrams. The difference occurs between cracking and yield. In this zone, the true shape of the curve is slightly higher than the idealized curve. The error between the two curves is about 1%, and since the idealized curve remains below the true curve, it is conservative to use the idealized curve. For heavily reinforced walls, the reinforcing steel would not reach its yield strength before the maximum amount of compression masonry was used. In this case, the diagram consisted of two lines connecting zero moment, cracking moment, and ultimate moment and curvatures.

The cracking moment was calculated by assuming that the tension masonry could crack without any axial load applied. If enough out-of-plane load was applied before the axial load could be applied, such as in a heavy wind event during construction, the masonry would crack and that strength could not be recovered. If this assumption is not used and heavy axial loads are applied, such as the ones used here, the moment required

to induce cracking of the tension masonry is larger than the yield moment, or in some cases, even the ultimate moment. Therefore, once the masonry cracks the wall surpasses the ultimate moment and causes failure. Using the assumption stated earlier allows for each wall to progress through cracking and on to ultimate. The cracking moment is taken as the gross section modulus times the modulus of rupture. The modulus of rupture was obtained from ACI 530-05 in Table 3.1.8.2.1 (Masonry Standards Joint Committee, 2004).

The yield moment is the moment necessary to cause the reinforcing steel to just reach the yield stress (for this analysis, grade 60 steel was used). Unlike the cracking moment, the yield moment is assumed to occur after the axial load has been applied. This allows for heavily reinforced walls that do not reach the yield moment before reaching their respective ultimate moment to consider the effects of moment magnification. This is a reasonable assumption since the yield moment is typically much higher than the cracking moment even for walls without axial loads. Statics dictates that the tension force equal the summation of the axial load and the compression masonry due to bending. With this in mind, if the axial load varies greatly, the neutral axis can vary throughout the cross section of the wall. To account for this, five possibilities exist and must be checked simultaneously to determine the true yield moment. Only one possibility will meet its respective conditions and produce the correct moment. Figures 4 through 8 show the five possibilities with their respective conditions.

As the strain in the masonry increases, the corresponding stress begins to “level off”. Once the strain reaches a value equivalent to eighty percent of the compressive strength divided by the modulus of elasticity, the stress has reached the maximum level

allowed by ACI 530-05. For simplicity, this strain will be referred to as the corresponding compressive strain (CCS). The value of the CCS is determined by the following relationship:

$$CCS = \frac{0.8f'_m}{E_m}$$

where f'_m is the compressive strength of masonry. ACI 530-05 defines E_m for concrete masonry units as $900f'_m$. The CCS for CMU then becomes 0.000889, or simply 0.8/900.

If the stress in the masonry reaches the CCS , the shape of the stress diagram in compression changes from a triangle to a trapezoid. This is why multiple possibilities occur for the value of the yield moment. Figures 4 through 8 show cases 1 through 5, respectively, along with the conditions that define that case and the equations for solving for the position of the neutral axis (kd) with respect to the extreme compression fiber of the cross-section. The equations for Cases 1, 3, and 5 are broken into the three terms of the quadratic equation (a , b , and c) as shown in the following equation:

$$kd = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

Case 1 occurs when the compressive strain is less than or equal to the CCS and the distance from the extreme compressive fiber to the neutral axis (d) is less than the thickness of the flange (t_f). In this case, the shape of the compressive stress block is simply a triangle and since the entire triangle is within the flange, the corresponding moment is a direct calculation. Case 2 occurs when the compressive strain is greater than the CCS and the distance to the neutral axis is less than the thickness of the flange. The compressive stress block then becomes a trapezoid and the yield moment must be

calculated accordingly. In Case 3, the compressive strain is greater than the *CCS*, the neutral axis falls in the web of the cross-section, and the point of inflection of the trapezoid falls in the flange. The yield moment then becomes a combination of the portion of the stress block in the web and the portion in the flange. In Case 4, the compressive strain is larger than the *CCS* and both the neutral axis and point of inflection lie within the web of the cross-section. This is the case when the amount of reinforcing steel is very large. In Case 5, the compressive strain is less than the *CCS* and the neutral axis lies within the web of the cross-section. For the wall thicknesses considered for this analysis, this case is only possible in 12 inch CMU walls.

The curve defining the true shape between cracking and yielding was developed using the same equations as those used for determining the yield moment and curvature. The stress in the steel was determined by establishing a point between the cracking stress and the yield stress. In all 10 points between cracking and yield were used to develop points along the true curve. The stress was then used to determine the force in the steel and thusly the area of compression masonry required to balance the tension force and the axial load. The one case that meets all the required conditions is then used to determine the moment and curvature for that point.

Since statics dictates that the summation of the axial force and the compressive force due to bending equal the tension force in the steel, equations had to be set up in the spreadsheet to solve for both the location of the neutral axis and the corresponding yield moment. For Cases 1, 3, and 5, the result is a quadratic equation, and for Cases 2 and 4, the result was an equality. Each equation runs simultaneously and the spreadsheet

determines, by a series of equality checks, which possibility is correct and returns the correct yield moment and corresponding curvature.

The ultimate moment was calculated as required by ACI 350-05. This method uses an equivalent rectangular stress block for the compression masonry. The equation is as follows:

$$\phi M_n = \phi(A_s f_y + P)(d - \frac{a}{2})$$

where A_s is the area of tension steel, f_y is the yield strength of steel, and P is the applied axial load. Normally, the steel will surpass its yield strain resulting in a straightforward calculation of the ultimate moment and curvature. For walls with large amounts of reinforcing steel, the depth (a) of the rectangular stress block may fall into the web of the cross-section causing a separate equation to have to be checked:

$$M_n = bt_f f'_m \left(d - \frac{t_f}{2} \right) + \left[(A_s f_y - bt_f f'_m) \left(d - t_f - \frac{a - t_f}{2} \right) \right]$$

The spreadsheet must then determine which equation is correct and return the true ultimate moment. In cases where the reinforcing steel is very large, the steel may not yield before the ultimate moment is reached. When this occurs, the actual stress in the steel must be calculated. The “Goal Seek” function in Microsoft Excel was incorporated into a macro to produce the true ultimate moment. Excel warns the user that the steel has not yielded and that the macro must be used. Once the macro is initiated, Excel changes the strain in the steel until the force in the steel matches the summation of the axial force and the compressive force due to bending. The following equation is used in the macro:

$$E_s = \frac{\left[\frac{0.8d \epsilon_m}{\epsilon_{mu} + \epsilon_s} - t_f \right] (0.8f'_m t_w - P + 0.8f'_m b t_f)}{A_s \epsilon_s}$$

Statics is met when this ratio results in the modulus of elasticity of steel ($E_s = 29000$ ksi).

By definition, the modulus of elasticity is stress divided by strain.

The magnitude of the axial load placed on a wall will also have an effect on the shape of the moment-curvature diagram. Figure 9 shows moment-curvature diagrams for walls with a nominal thickness of 8 inches with #4 bars at 48 inch spacing and axial loads of 5, 10, 15 and 20% $f'_m A_n$. The yield and axial moments increase as the axial load increases. However, the curvature between yield and ultimate decreases substantially. This is a good indication of a wall's ductility. In effect, increased axial load results in a larger ultimate moment, but reduced ductility.

Once the moments and curvatures have been determined, a list of these parameters is needed in a manner that can be used in the finite element model. Since the model will not have any sort of geometric dimensions other than height, it makes sense to produce a moment-curvature diagram based on a per unit basis. To accomplish this, all moments are then divided by the spacing of the reinforcing steel. The result is a list of moments on a per foot basis for the ordinate and their corresponding curvatures for the abscissa.

3. Model and Modeling Techniques

Once the wall behavior has been defined via the moment-curvature diagram, this information must be placed into an idealized model of the wall. Rather than utilize modeling software with built in structural engineering assumptions, a package was needed that was capable of loading a model without predefined materials. ADINA provides the capability of using a moment-curvature diagram to establish the material properties of the wall. ADINA also is also capable of non-linear analyses, which in turn gives a more accurate interpretation of real world phenomena (ADINA, 2005).

Rather than model the entire length, the wall was modeled on a per foot basis. With so many models to consider, having a “template” for each height of wall resulted in quicker modeling times. Each wall is idealized as a beam spanning vertically as shown in Figure 10. The bottom of the wall is considered pinned and a roller supports the top of the wall. If a pin supports the top, no axial load is transferred into the wall. ADINA considers a pin to have axial restraint, so the axial load added to the top of the wall would be taken out at the support. Each model was analyzed assuming large displacements and $P-\Delta$ effects.

The loads that are applied to the wall are a concentrated axial load and a uniform out-of-plane load. The axial load is held static throughout the running of the model and the out-of-plane load increases until failure. ADINA provides a time-step function that allows the user to increase a load with time. For the models used here, the out-of-plane load is increased at a rate of one tenth of a pound per time unit.

Each model is divided into twenty subdivided lines before meshing takes place. The results of the model is more accurate with an increasing number of subdivided lines. Other engineering properties of the wall are placed into the model such as torsional resistance, in-plane bending resistance, out-of-plane bending resistance (defined by the moment-curvature spreadsheet), and the axial force vs. strain curve. All of these parameters are constant from model to model except the out-of plane bending. Since the model is loaded in two directions and bending takes place in one direction, the development of the other parameters is not as critical as the out-of-plane moment-curvature diagram.

In order to insure that a wide range of h/r ratios is considered, four heights of 12in. CMU walls and five heights of 8in. CMU walls were modeled. Both wall thicknesses were modeled with 12, 16, 20, and 24-foot heights and the 8-inch CMU walls included 8-foot wall heights. In all, 180 walls were modeled.

After all walls had been modeled and analyzed, the moment magnification for each wall was calculated. ADINA transfers the uniform load into nodal loads. The solution that results is actually the solution returned by analyzing the beam with these nodal loads. The result of the analysis becomes more accurate as the number of subdivided lines, and therefore nodal loads, increases. Most finite element programs then provide a correction for the solution by adding the fixed-end moments back into the moment resulting from the nodal loads. ADINA does not provide this correction. In order to obtain the true moment, the fixed-end moment is added to the resulting moment ADINA outputs. Had this correction not have taken place, the error resulting from a model with 20 subdivisions is still small. Since moments are proportional to length

squared, the error is on the order of $(1/20)^2$, or less than one-quarter of one percent. The moment magnification is then calculated by dividing the resulting moment, with the correction, by the static moment.

4. Results

Figures 11 and 12 show how the walls behave as the out-of-plane load is increased. Figure 11 shows four P- Δ curves for heavily reinforced walls. The ductility of the wall can be seen by the large amount of deflection the wall undergoes once the wall has cracked. As the axial load increases, the total deflection decreases. This has more to do with the ACI 530-05 definition of the ultimate moment rather than the increase in axial load. As the axial load increases, the ultimate moment decreases. This exchange of axial load versus ultimate moment in this case results in smaller deflections with larger axial loads. In Figure 12 the ductility is much less due to the small amount of reinforcing. Once the wall cracks and the steel resists the tension, the reinforcing is only able to experience a small strain increase, relative to the heavier reinforced walls, before reaching ultimate. The magnitudes of the displacement in this case increase as the axial load increases. This behavior is more typical of the effect increased axial load has on deflection. The increased axial load will induce a larger moment on the wall as the axial load becomes eccentric.

Figures 13 through 22 show how the moment magnification increases with increasing h/r ratios for both 8in. and 12in. CMU walls. However, once this ratio approaches 75, the moment magnification begins to drop. This decrease is due to the walls not having sufficient stiffness to resist large axial loads with eccentricity. Once the out-of-plane load causes a small amount of eccentricity, the wall fails due to the large moment caused by the simultaneous loading. Using the same criteria as stated earlier, the

limiting h/r ratio for which no moment magnification needs to be considered (i.e. moment magnification is less than five percent) is around 10. This corresponds to an 8" CMU wall with a height of just over two feet and a 12 inch CMU wall just less than four feet high. The h/r ratio for which the moment is increased by 10% is 19. This is an 8" CMU wall four feet high and a 12" CMU wall 7'-3" tall. All other walls would have to consider moment magnification.

These requirements are not reasonable. Any wall used for building purposes would be eight feet tall at a minimum. Any shorter and the wall would have little or no structural purpose. However, since the existing requirements in ACI 530-05 distinguish between walls with an axial stress of five percent of the walls net area times the compressive strength of the masonry, it seems reasonable to do so here as well. Walls with this amount of axial stress or less would not have to consider moment magnification if the h/r ratio is less than 24. This corresponds to an 8" CMU wall about five feet tall and a 12" CMU wall about ten feet tall. Walls that would increase the applied moment by 10% would have h/r ratios between 24 and 40. This would include 8" CMU walls between five and nine feet tall and 12" CMU walls between nine and fifteen feet tall. All walls with h/r ratios over 40 would have to consider the effects of moment magnification.

Another means of organizing moment magnification can be seen in Figures 23 through 27. These graphs show how moment magnification is affected by increased amounts of steel reinforcing. No general trend can be seen in these graphs. At best it is possible to summarize that for a given wall moment magnification increases as the area of steel increases for shorter walls. This relationship breaks down with taller walls.

5. Conclusion

This attempt to find practical limits between neglecting, using a simplified method, and fully considering the effects of moment magnification yielded poor results. The stiffness of an unreinforced masonry is much greater than that of a reinforced masonry wall. This seems counterintuitive, but the moment of inertia of the unreinforced masonry wall is larger since a larger area is considered. The moment of inertia of an unreinforced 8" CMU wall is 308.7 in^4 while the moment of inertia of a reinforced masonry wall, after cracking, is 67.5 in^4 . With this in mind, it is reasonable to assume that the method of limiting moment magnification calculations set forth by Dr. R. Bennett is not feasible for reinforced masonry walls.

Rather than allowing engineers to neglect moment magnification in reinforced masonry walls for small h/r ratios, moment magnification should be considered for all wall heights. For walls with h/r ratios less than 40, the static moment shall be increased by 10%. This would be the simplified method for considering moment magnification. All walls with h/r ratios greater than 40 shall abide by the current requirements of ACI 530.

List of References

List of References

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- ADINA: Automatic Dynamic Incremental Nonlinear Analysis*, Version 8.3.3. ADINA R&D, Inc., 2005.
- Bennett, R.M. Code proposal for unreinforced masonry design, Private Communication, 2006.
- Canadian Standards Association. *Masonry Design for Buildings (Limit States Design)*. S304.1-94, 1994.
- Horton, R.T. and M.K. Tadros. "Deflection of Reinforced Masonry Members." *ACI Structural Journal* 87.4 (1990): 453-63.
- MacGregor, J.G., J.E. Breen and E.O. Pfrang. "Design of Slender Concrete Columns." *ACI Journal*, Proceedings 67.1 (1970): 6-28.
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- Masonry Standards Joint Committee. *Building Code Requirements for Masonry Structures*. ACI 530-05, 2005.

Appendix

Table 1. Available Values for Variables in Spreadsheet

Variable	Values
Masonry type	<ul style="list-style-type: none"> • 6" CMU • 8"CMU • 10"CMU • 12"CMU
Steel	One piece of rebar ranging from a #3 to a #14 bar
Spacing	<ul style="list-style-type: none"> • 16" • 24" • 32" • 40" • 48"
Axial load	Input as a ratio of $f'_m \cdot A_n$
Mortar type	<ul style="list-style-type: none"> • Type S ($f'_m = 1500\text{psi}$) • Type N ($f'_m = 1350\text{ psi}$)

Table 2: Values Used for Variables in Spreadsheet

Variable	Values
Masonry type	<ul style="list-style-type: none"> • 8"CMU • 12"CMU
Steel and steel spacing	<p>For 8" CMU:</p> <ul style="list-style-type: none"> • #4 at 48" • #5 at 40" • #6 at 40" • #7 at 40" • #8 at 40"* <p>For 12" CMU:</p> <ul style="list-style-type: none"> • #4 at 32" • #5 at 24" • #6 at 24" • #7 at 24"* • #8 at 24"*
Axial load	<ul style="list-style-type: none"> • $0.05 * f'_m * A_n$ • $0.10 * f'_m * A_n$ • $0.15 * f'_m * A_n$ • $0.20 * f'_m * A_n$
Mortar type	Type N ($f'_m = 1350$ psi)

* denotes walls with reinforcing that exceeds the maximum allowed by ACI 530-05.

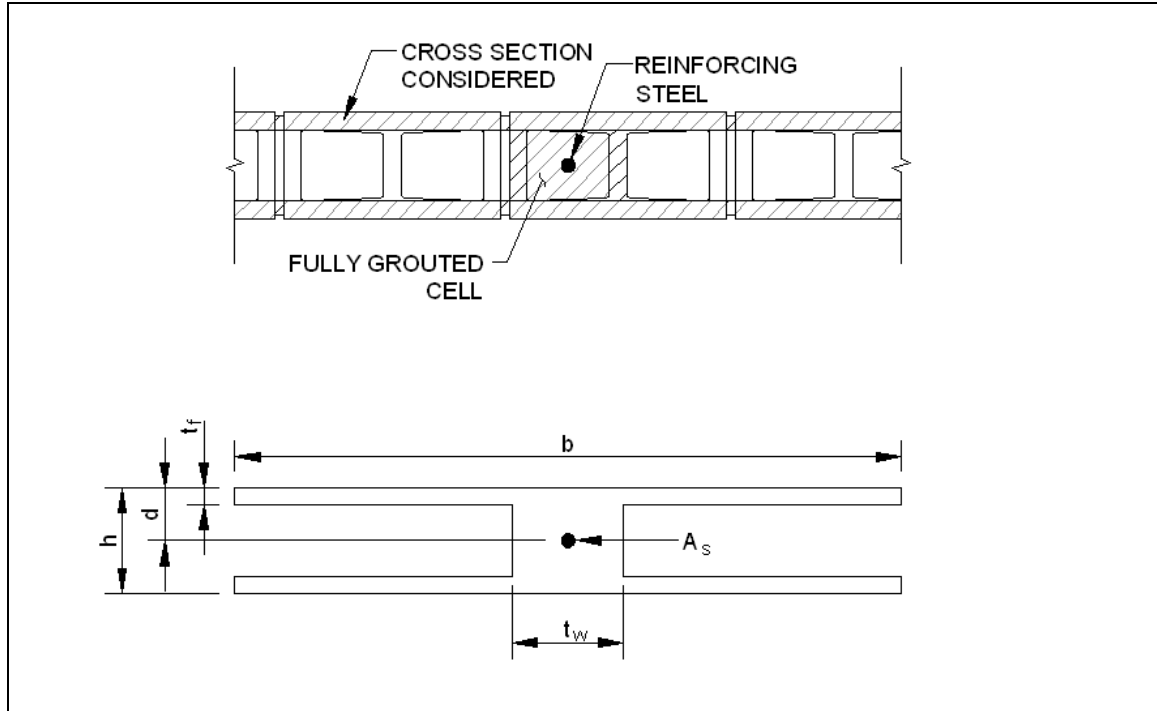


Figure 1. Cross-section of masonry wall (top) and idealized cross-section used in analysis (bottom)

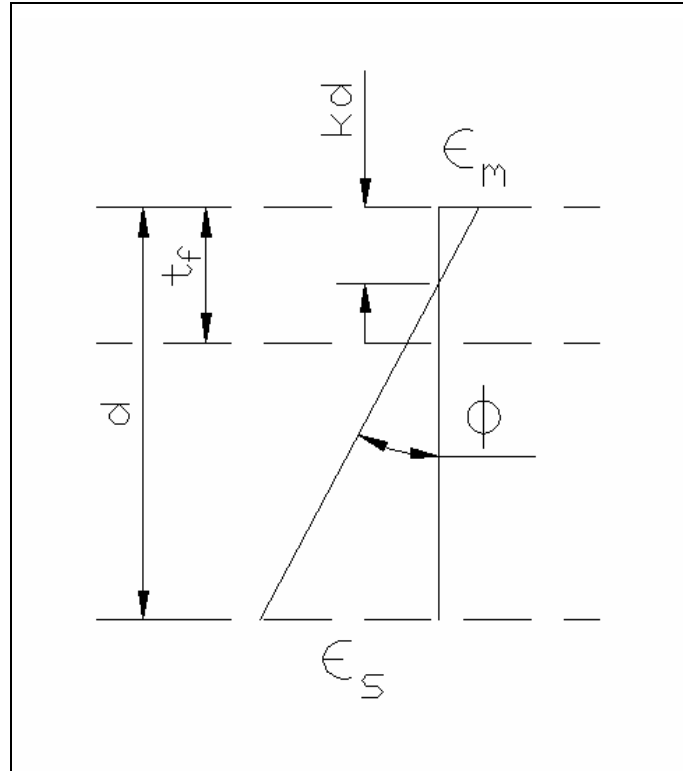


Figure 2. Strain diagram illustrating curvature (Φ)

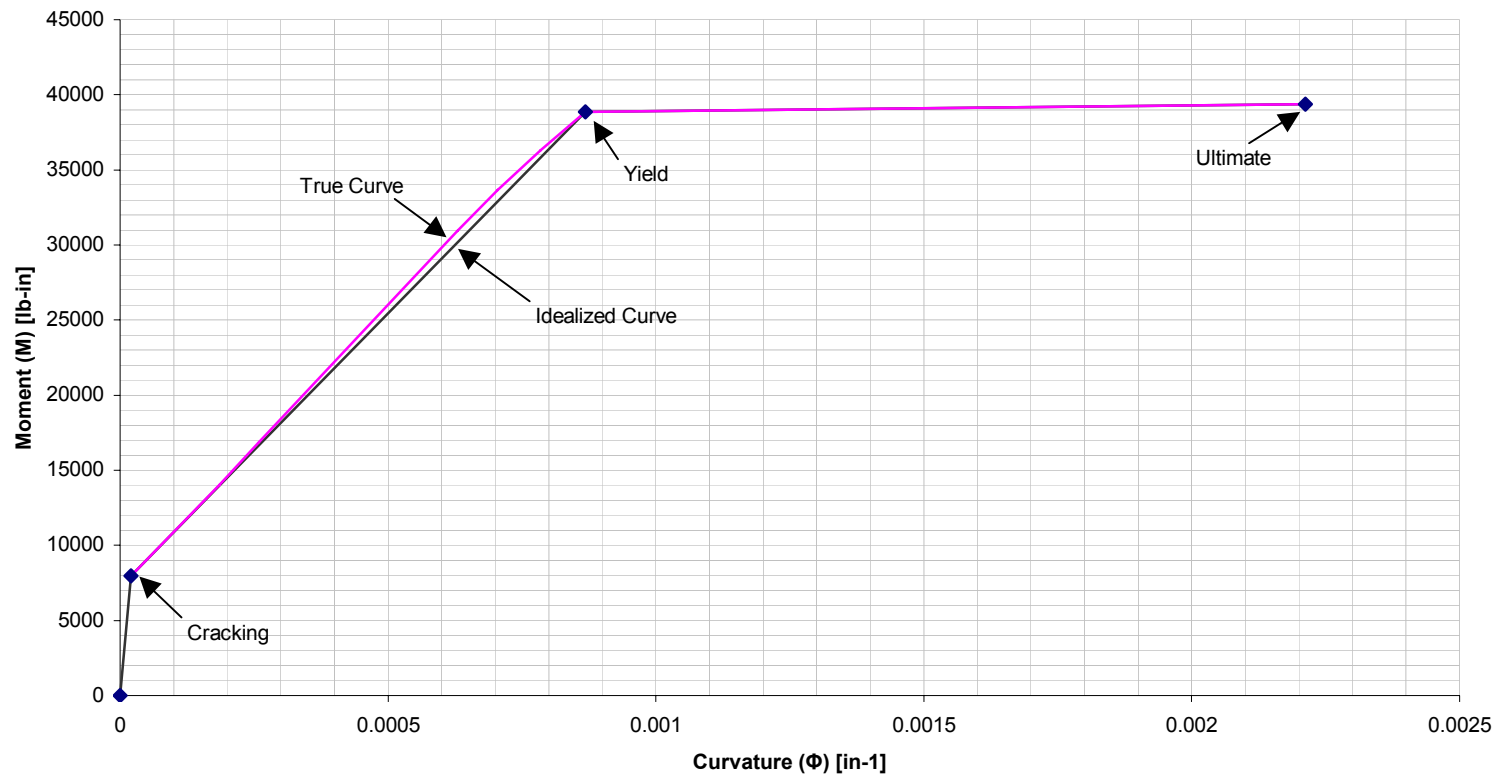
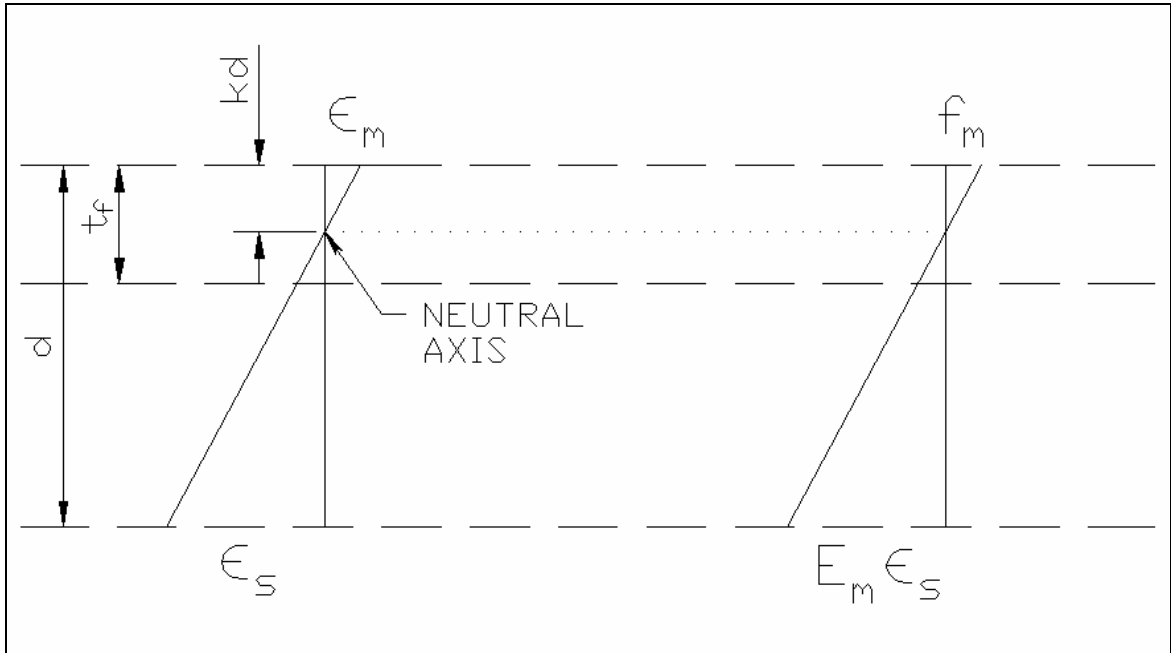


Figure 3. Typical moment-curvature diagram



Conditions:

$$\epsilon_m < \frac{0.8f'_m}{E_m}$$

$$kd < t_f$$

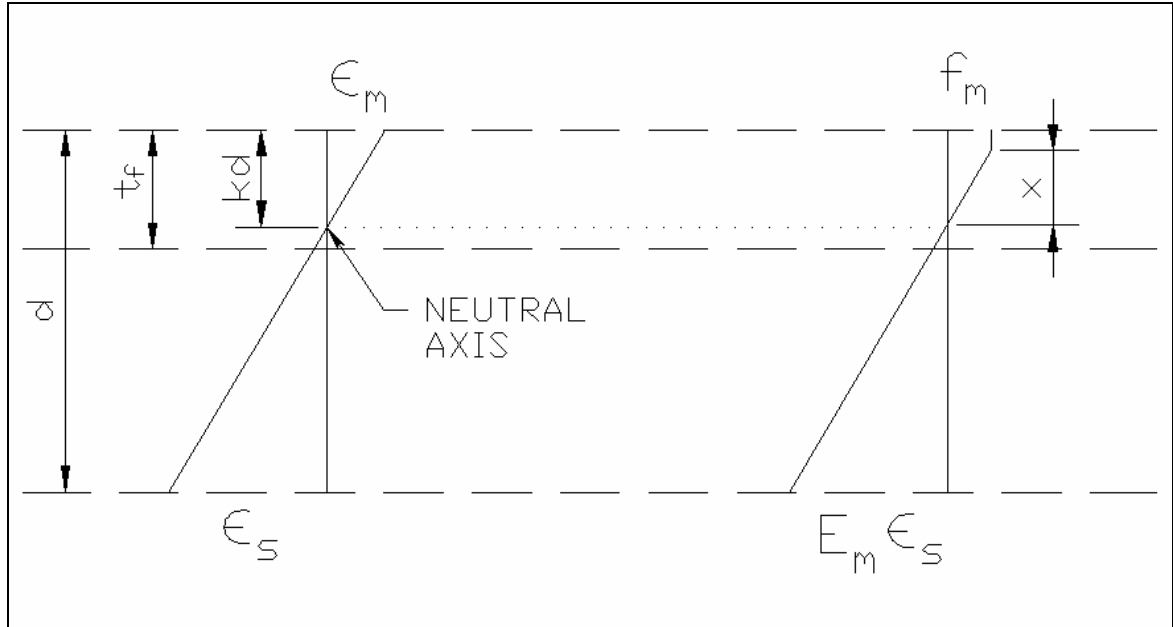
Quadratic terms:

$$a = \frac{1}{2} \epsilon_y E_m b$$

$$b = A_s f_y + P$$

$$c = -A_s f_y d - P$$

Figure 4. Case 1 for finding yield moment and curvature



Conditions:

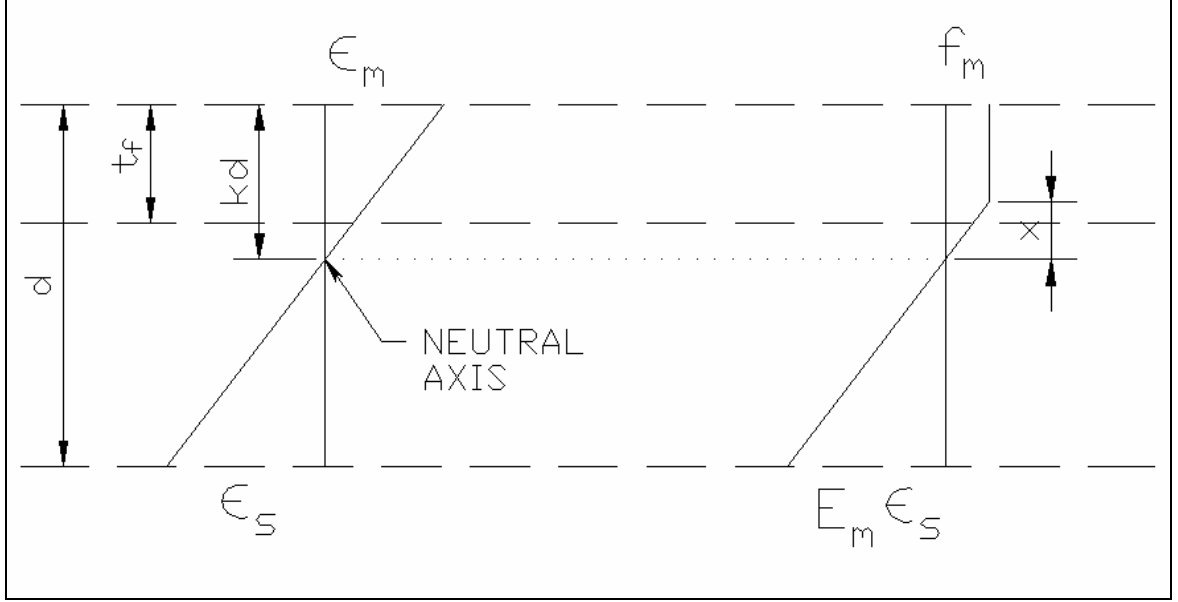
$$\epsilon_m > \frac{0.8f'_m}{E_m}$$

$$kd < t_f$$

Equation:

$$kd = \frac{A_s f_y + P + \frac{0.32 f_m'^2 b d}{\epsilon_y E_m}}{0.8 f_m' b + \frac{0.32 f_m'^2 b}{\epsilon_y E_m}}$$

Figure 5. Case 2 for finding yield moment and curvature



Conditions:

$$\varepsilon_m > \frac{0.8f'_m}{E_m}$$

$$kd > t_f$$

$$x > kd - t_f$$

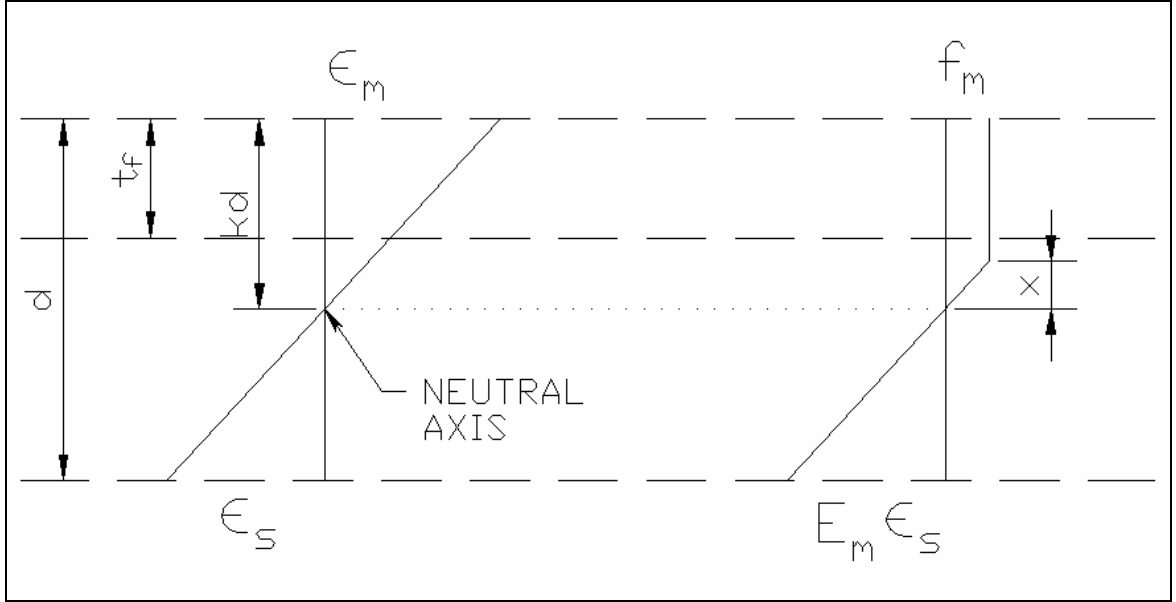
Quadratic terms:

$$a = -0.8f'_m b - \frac{0.32f_m'^2 b}{\epsilon_y E_m} - 0.5 \epsilon_y E_m b - 0.5 \epsilon_y E_m t_w$$

$$b = 0.8f'_m b d + \frac{0.64f_m'^2 b d}{\epsilon_y E_m} + \epsilon_y E_m b t_f + A_s f_y + P - \epsilon_y E_m t_f t_w$$

$$c = -\frac{0.32f_m'^2 b d^2}{\epsilon_y E_m} - 0.5 \epsilon_y E_m b t_f^2 - 0.4f'_m t_w t_f d - 0.5 \epsilon_y E_m t_w t_f^2 - A_s f_y d - P d + 0.5 \epsilon_y E_m t_w t_f^2$$

Figure 6. Case 3 for finding yield moment and curvature



Conditions:

$$\epsilon_m > \frac{0.8f'_m}{E_m}$$

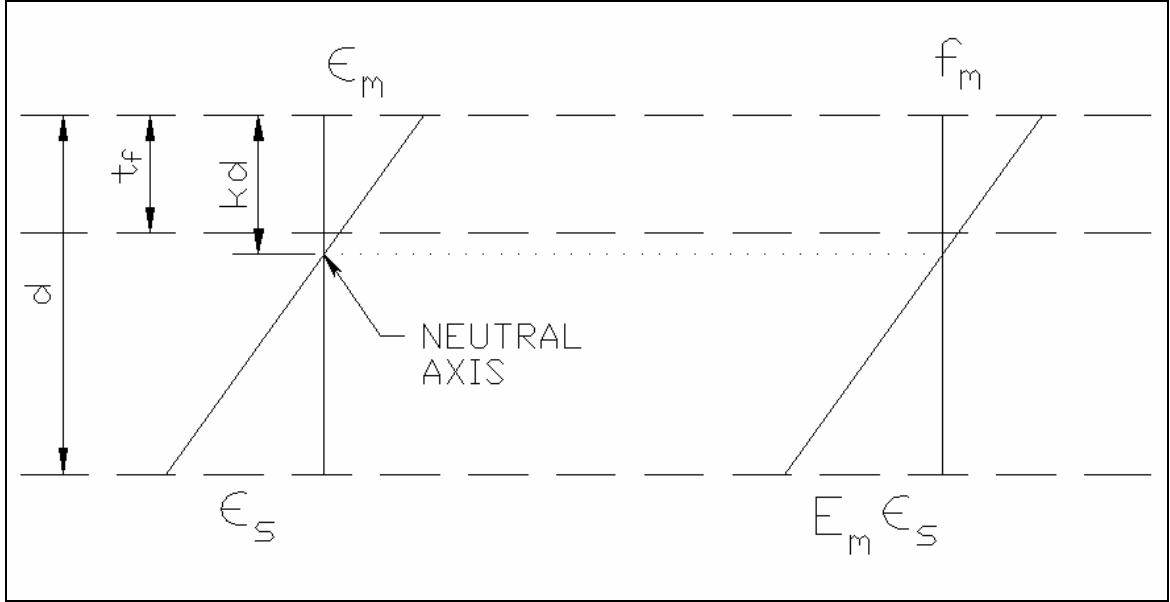
$$kd > t_f$$

$$x > kd - t_f$$

Equation:

$$kd = \frac{A_s f_y + P - 0.8f'_m b t_f + 0.8f'_m t_w t_f + \frac{0.32f_m'^2 t_w d}{\epsilon_y E_m}}{0.8f'_m t_w + \frac{0.32f_m'^2 t_w}{\epsilon_y E_m}}$$

Figure 7. Case 4 for finding yield moment and curvature



Conditions:

$$\varepsilon_m < \frac{0.8f'_m}{E_m}$$

$$kd > t_f$$

Quadratic terms:

$$a = 0.5 \varepsilon_y E_m t_w$$

$$b = \varepsilon_y E_m b t_f - \varepsilon_y E_m t_w t_f + A_s f_y + P$$

$$c = -0.5 \varepsilon_y E_m b t_f^2 + 0.5 \varepsilon_y E_m t_w t_f^2 - A_s f_y d - P$$

Figure 8. Case 5 for finding yield moment and curvature

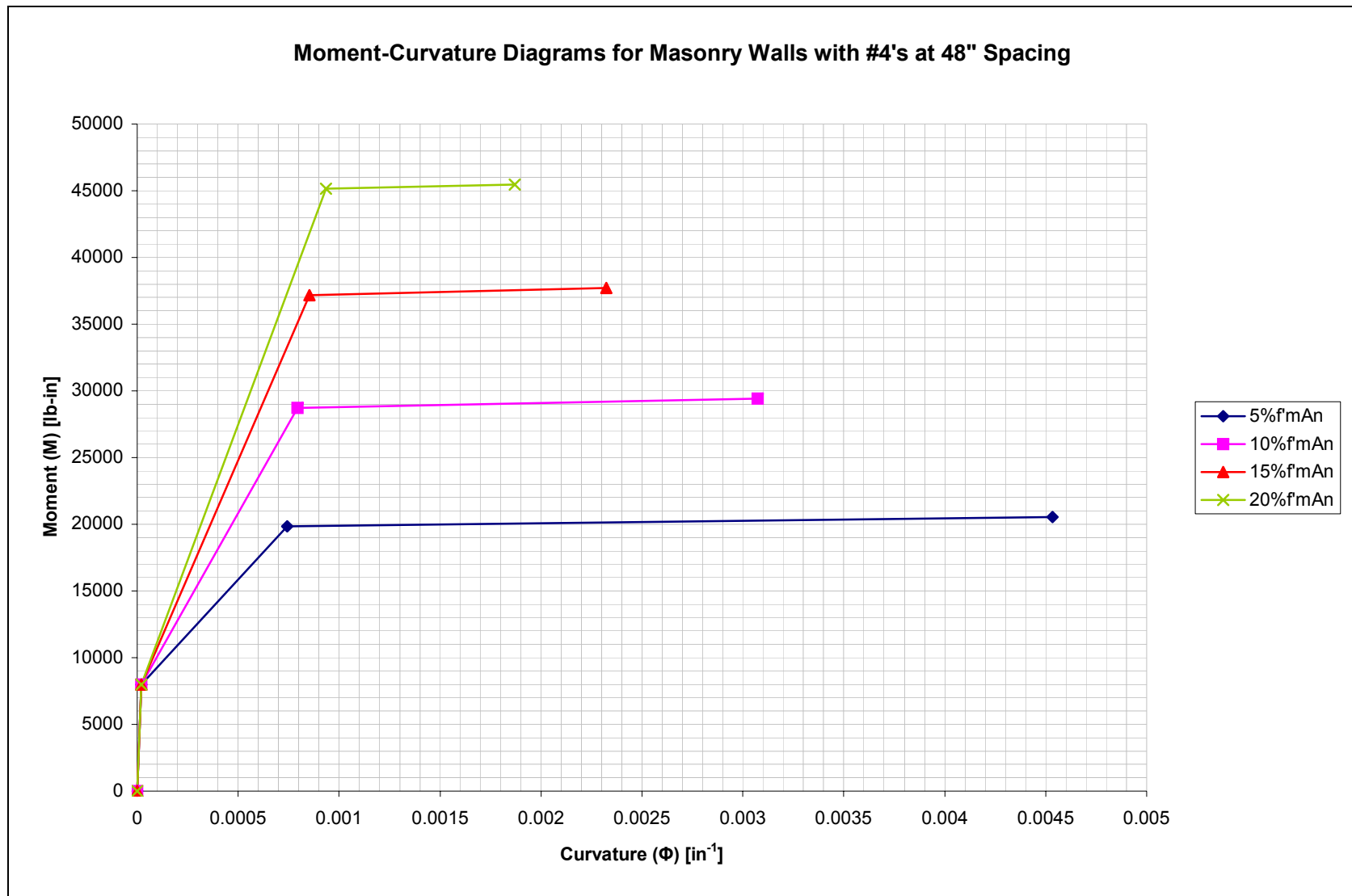


Figure 9. Moment-Curvature Diagrams for Masonry Walls with #4's at 48" Spacing

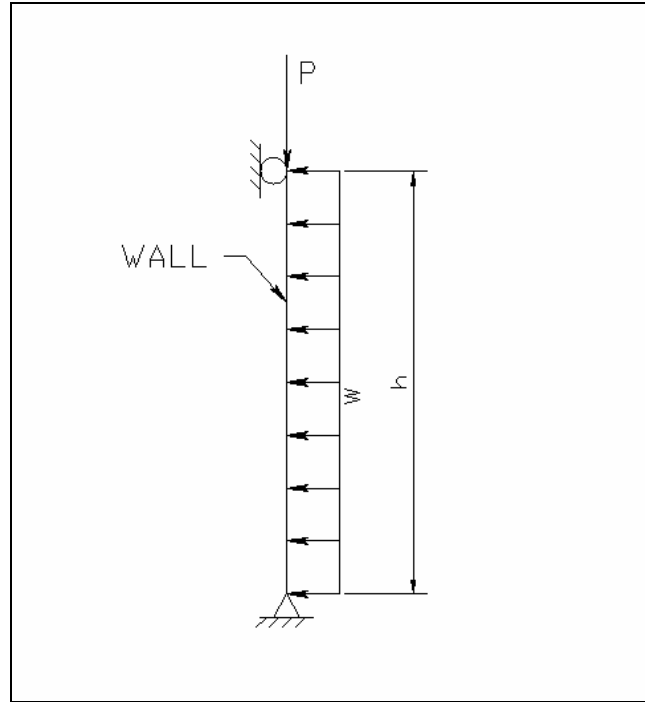


Figure 10. Free body diagram of loaded wall used in model

P-delta curves for heavily reinforced 16ft high walls 8in nominal thickness

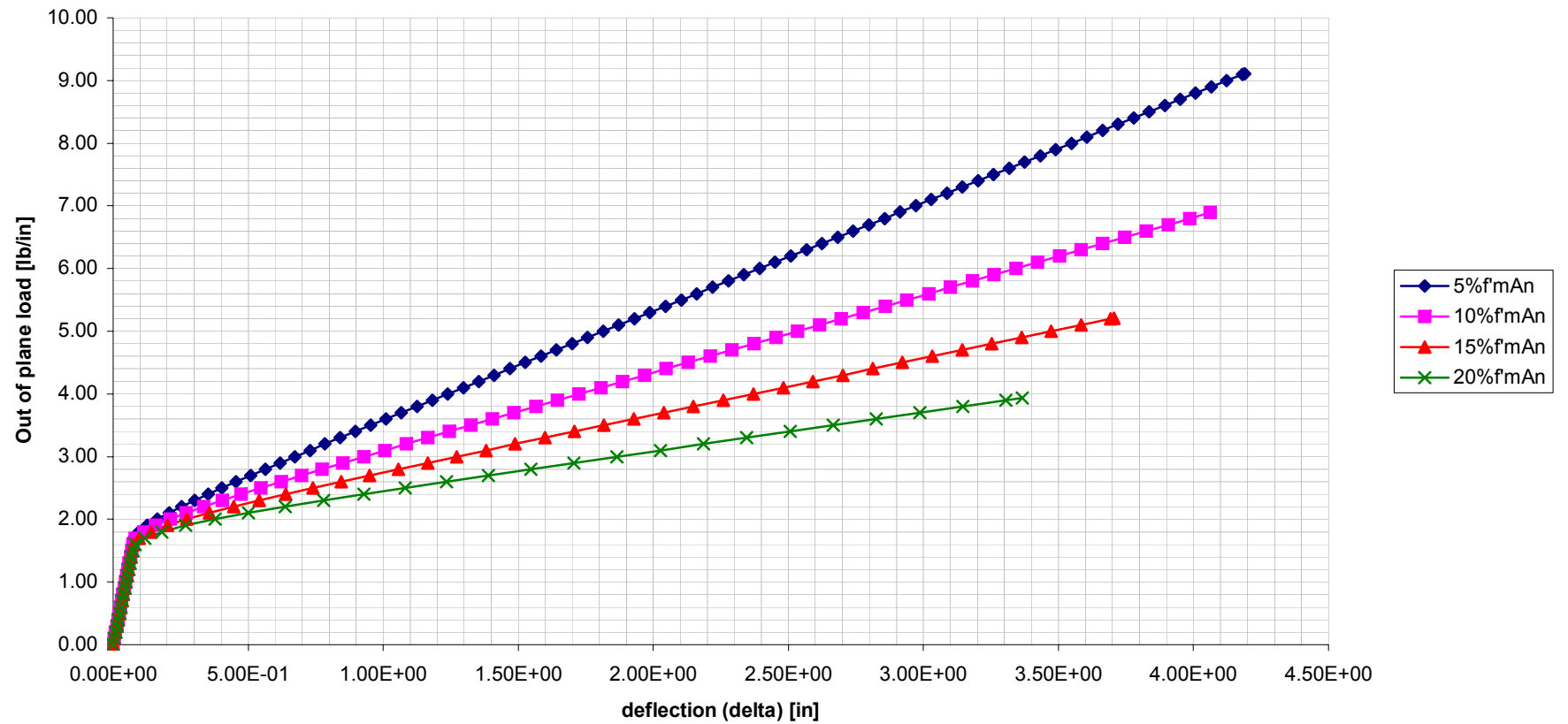


Figure 11: P- Δ curves for 8" thick, 16ft high walls with heavy reinforcing (#8's at 40 inches)

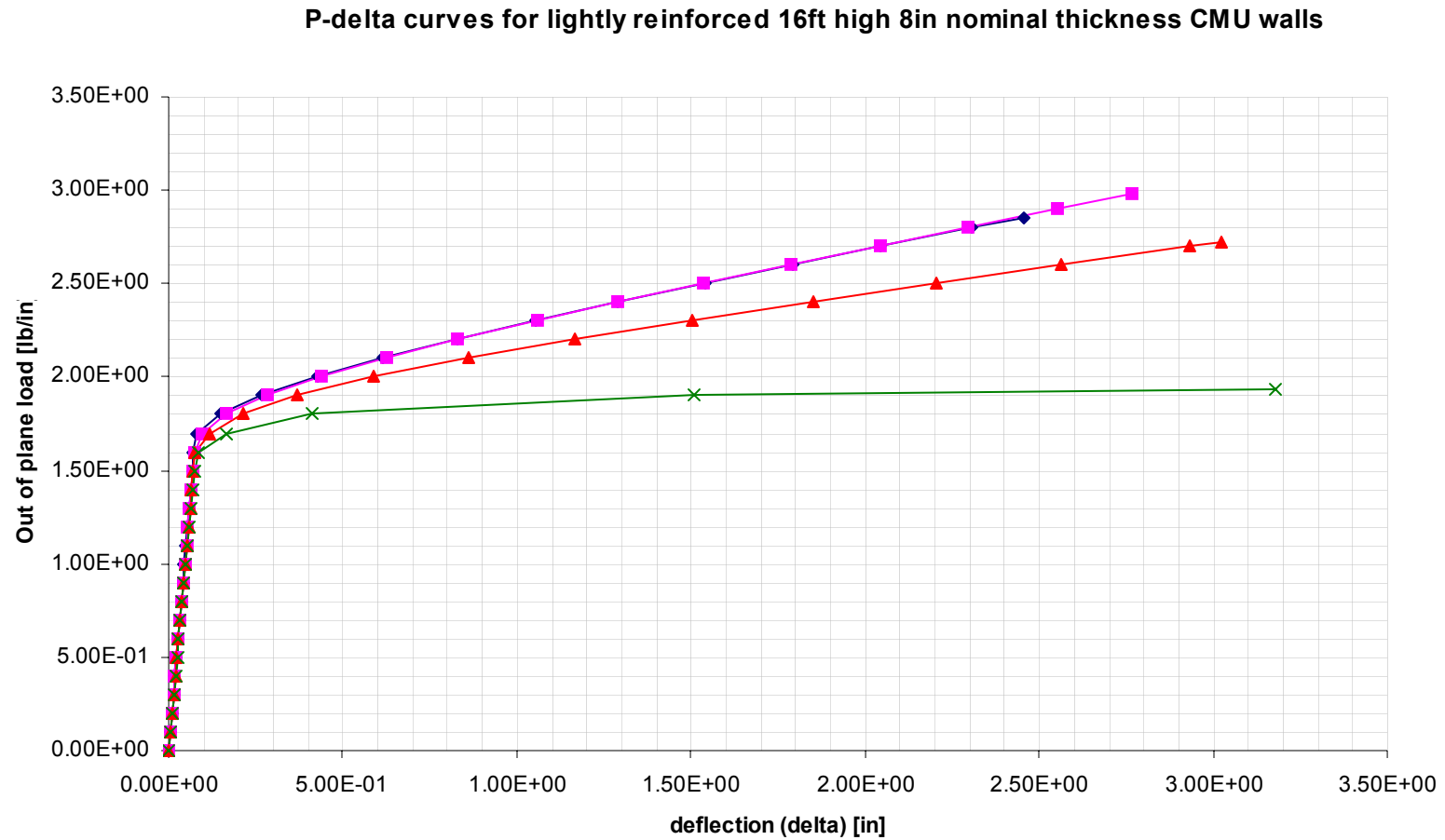


Figure 12: P- Δ curves for 8" thick, 16ft high walls with light reinforcing (#4's at 48 inches)

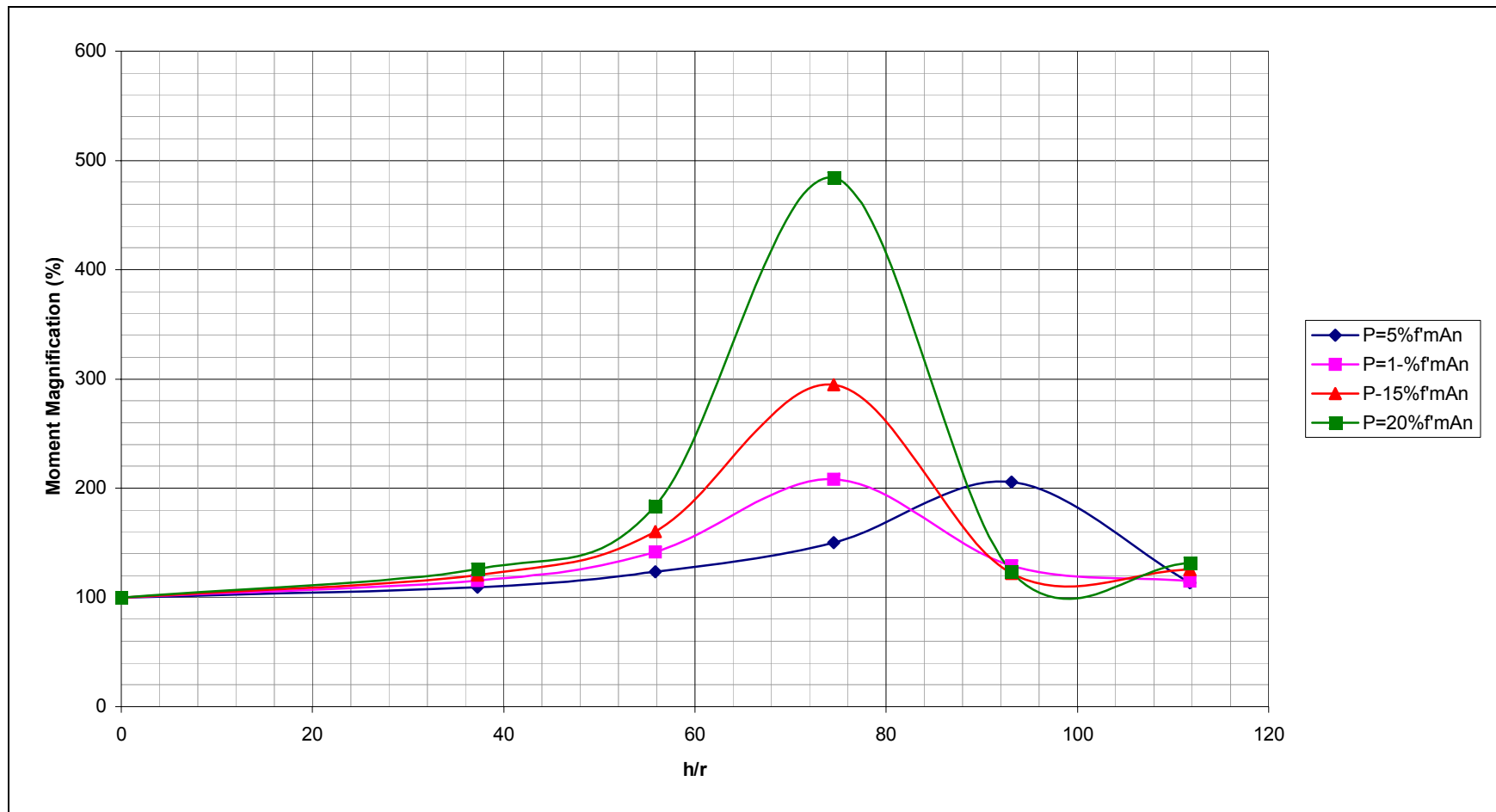


Figure 13. Moment Magnification vs. h/r ratio for 8"CMU, #4@48"

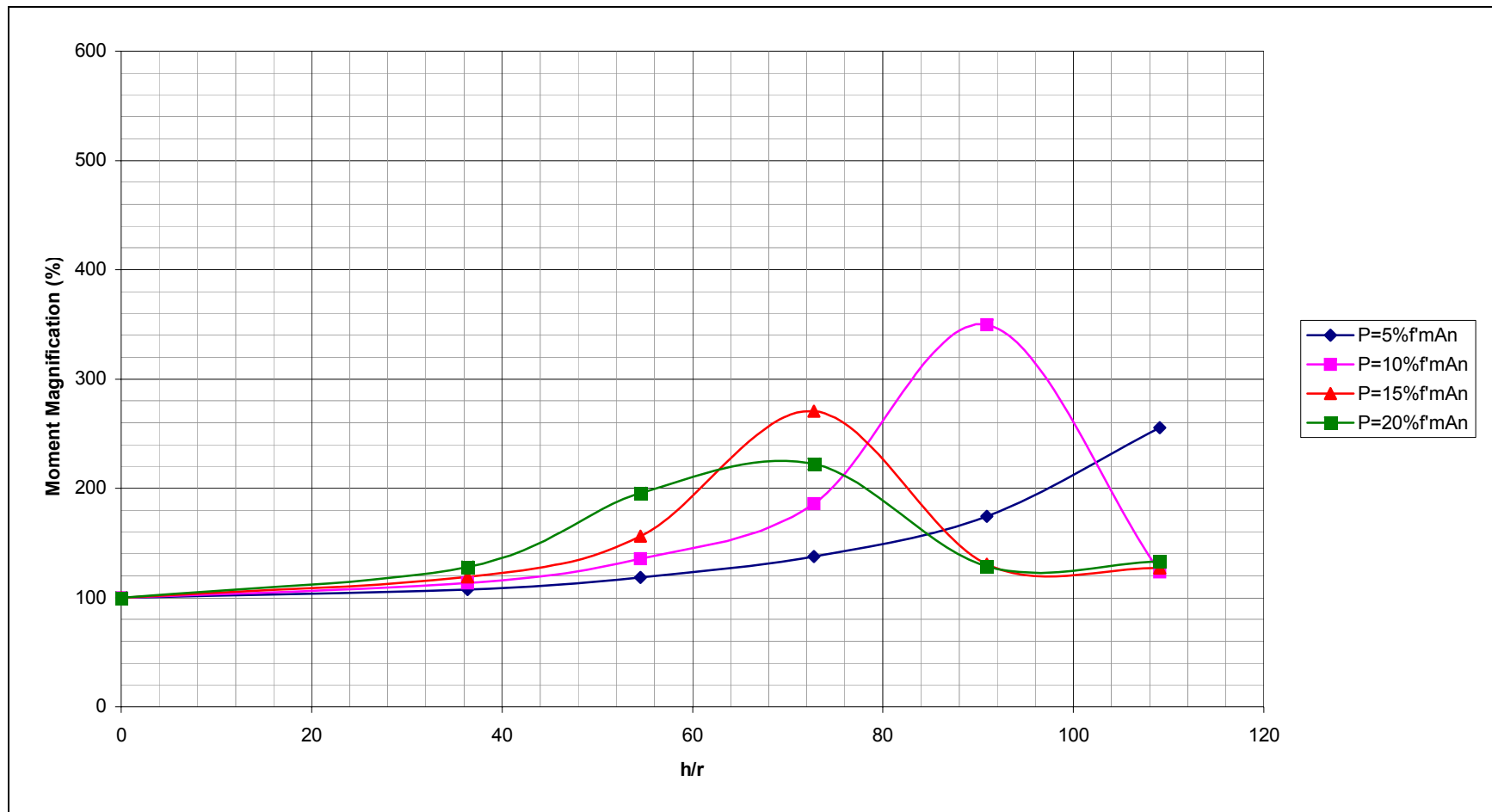


Figure 14. Moment Magnification vs. h/r ratio for 8"CMU, #5@40"

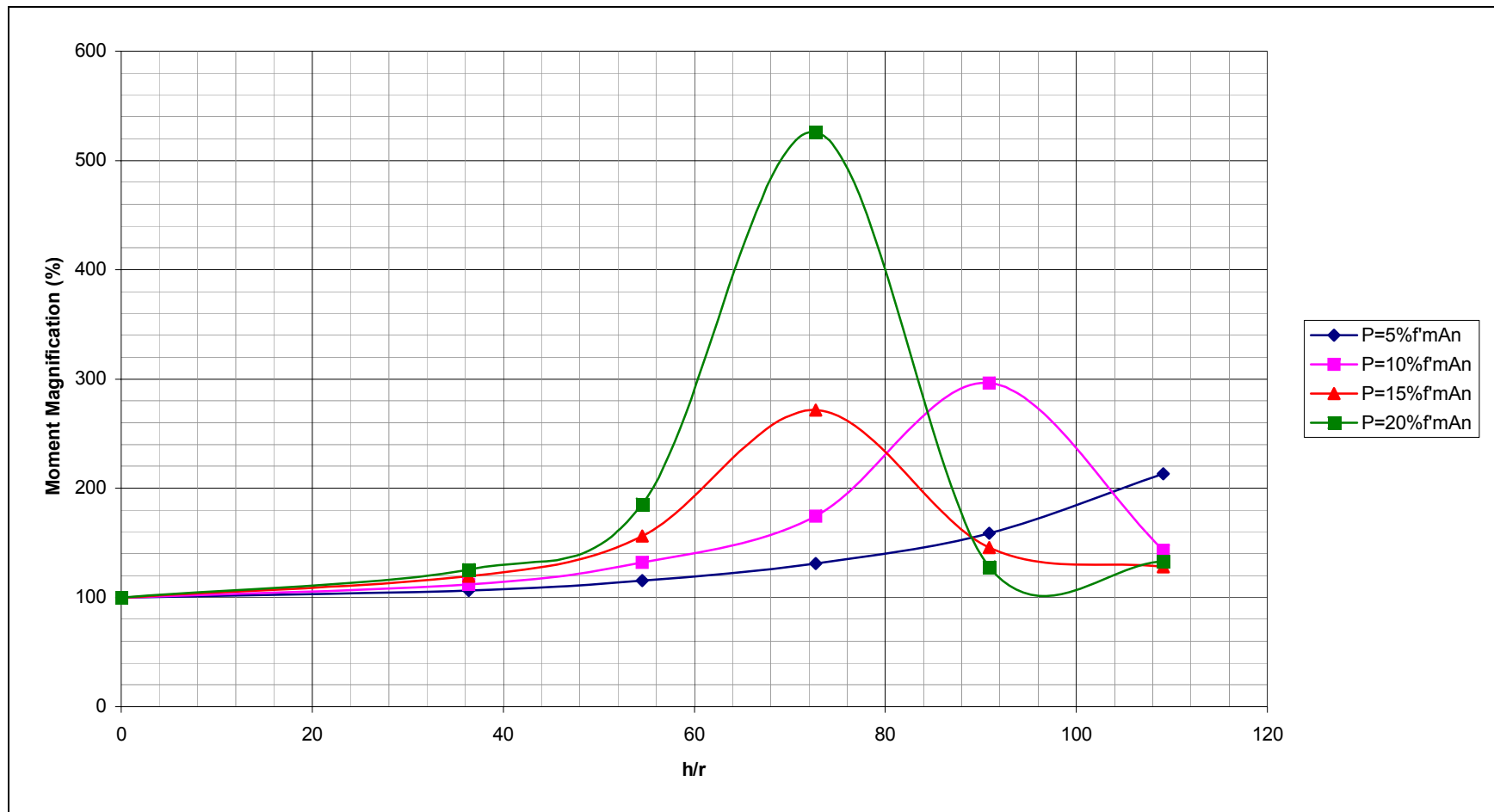


Figure 15. Moment Magnification vs. h/r ratio for 8"CMU, #6@40"

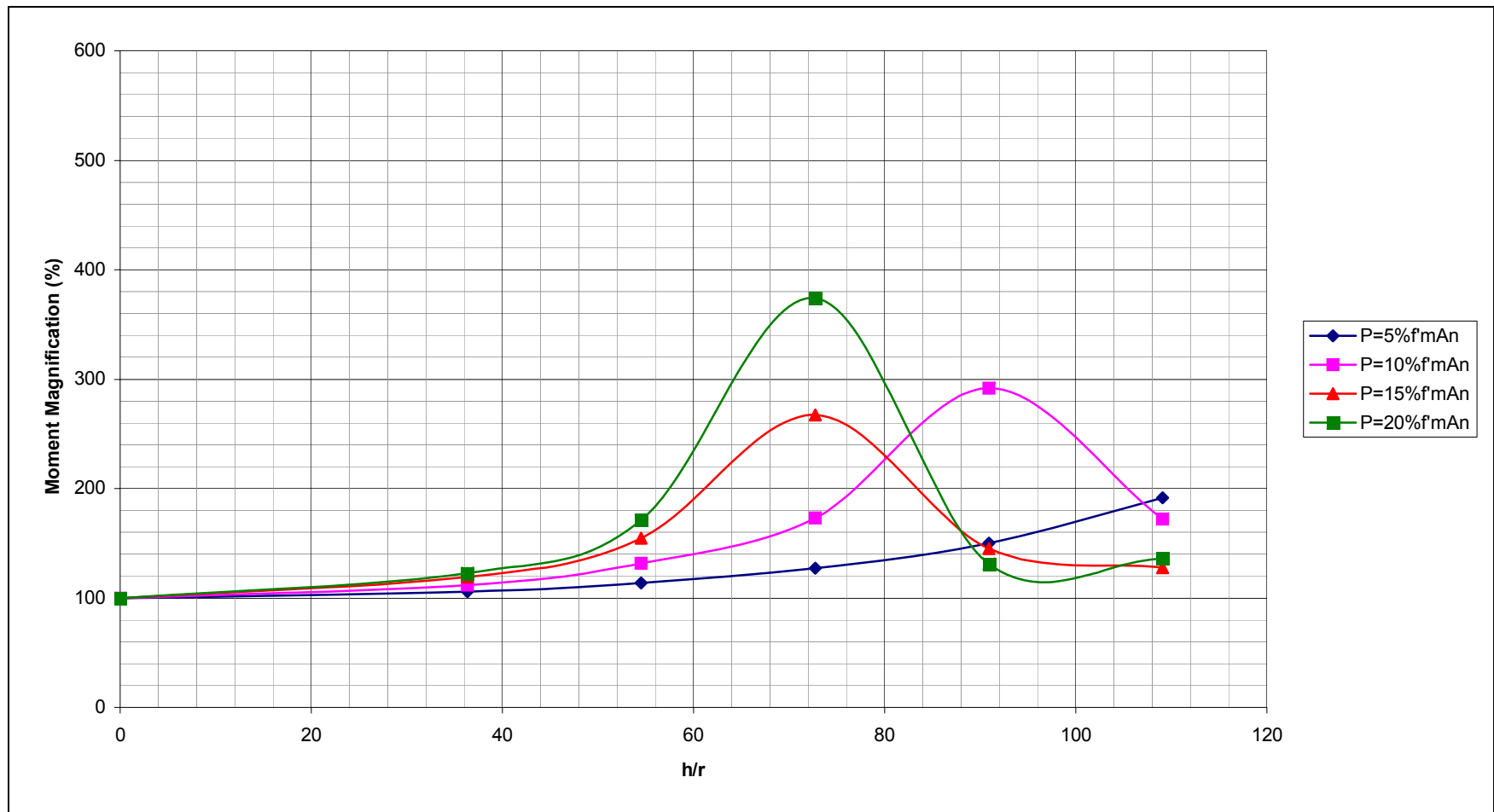


Figure 16. Moment Magnification vs. h/r ratio for 8"CMU, #7@40"

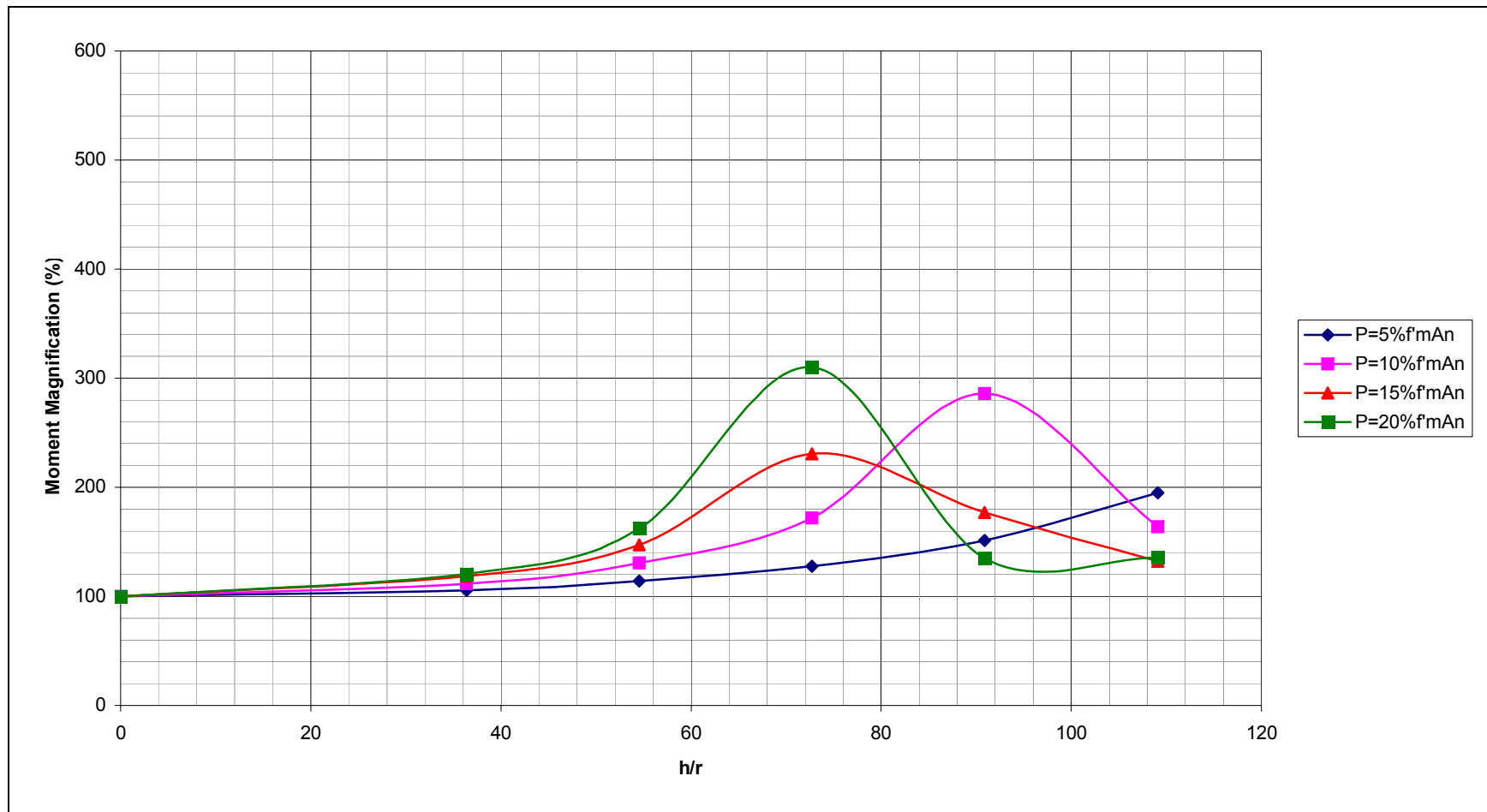


Figure 17. Moment Magnification vs. h/r ratio for 8"CMU, #8@40"

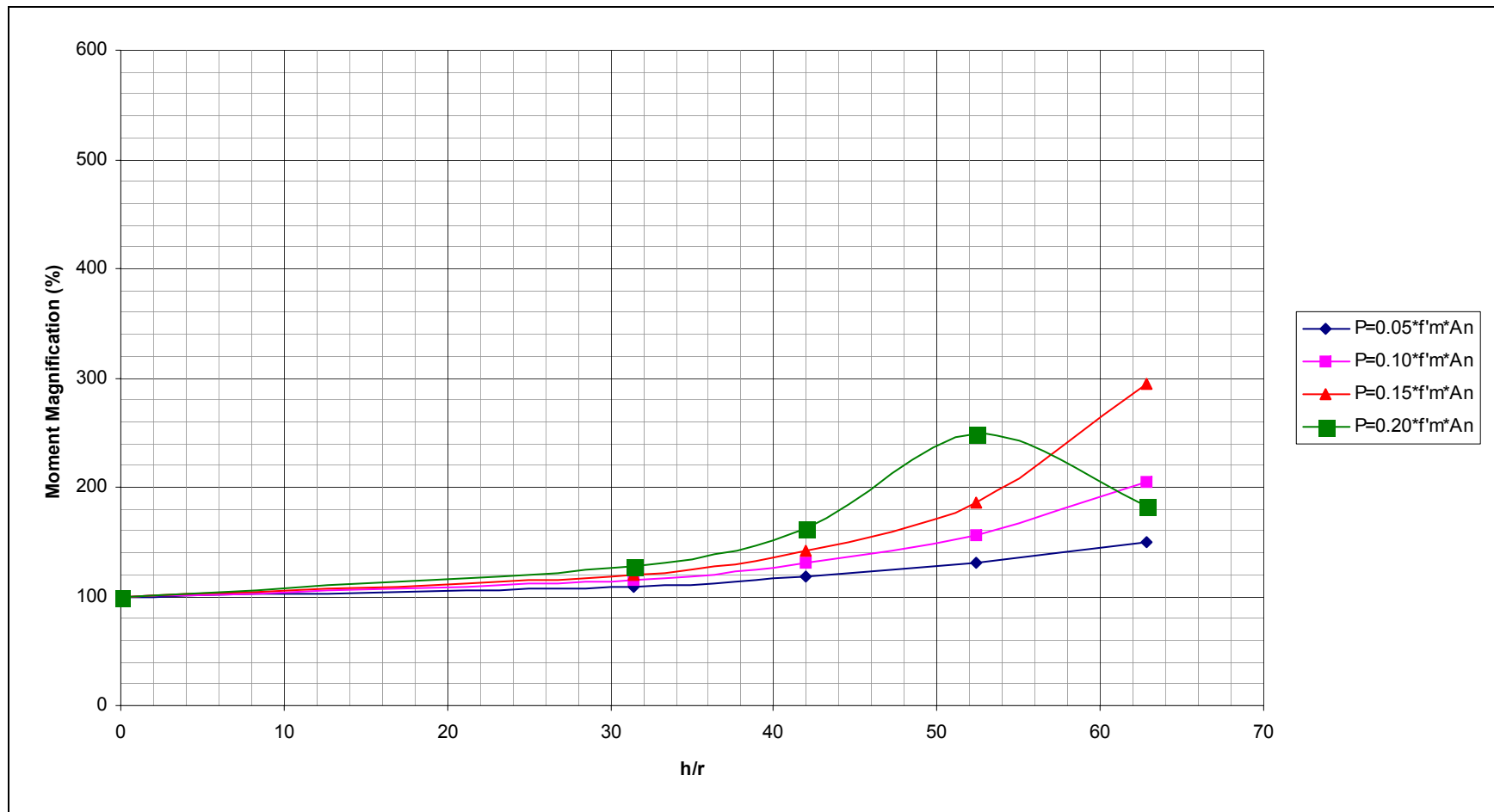


Figure 18. Moment Magnification vs. h/r ratio for 12''CMU, #4@32''

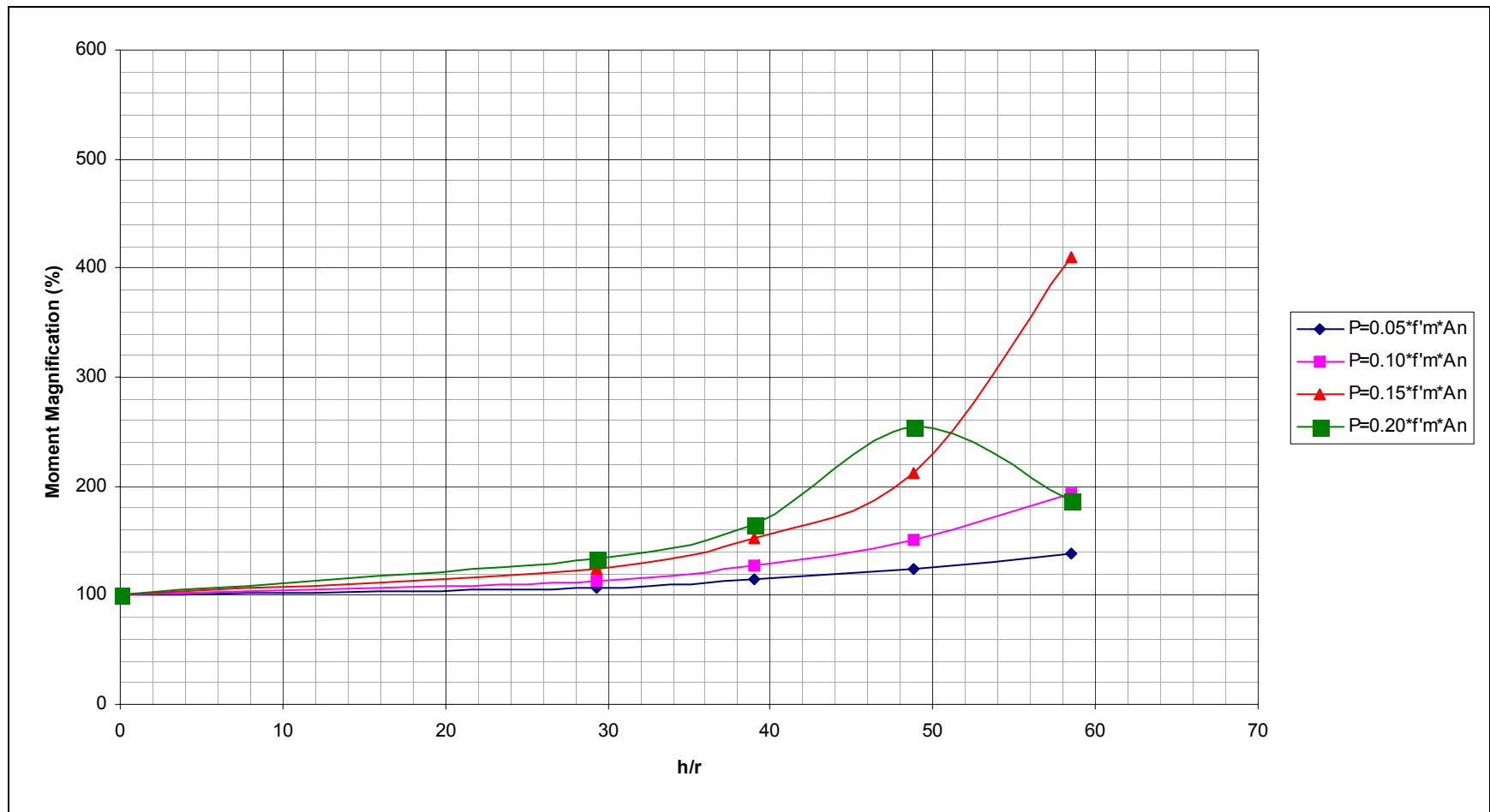


Figure 19. Moment Magnification vs. h/r ratio for 12"CMU, #5@24"

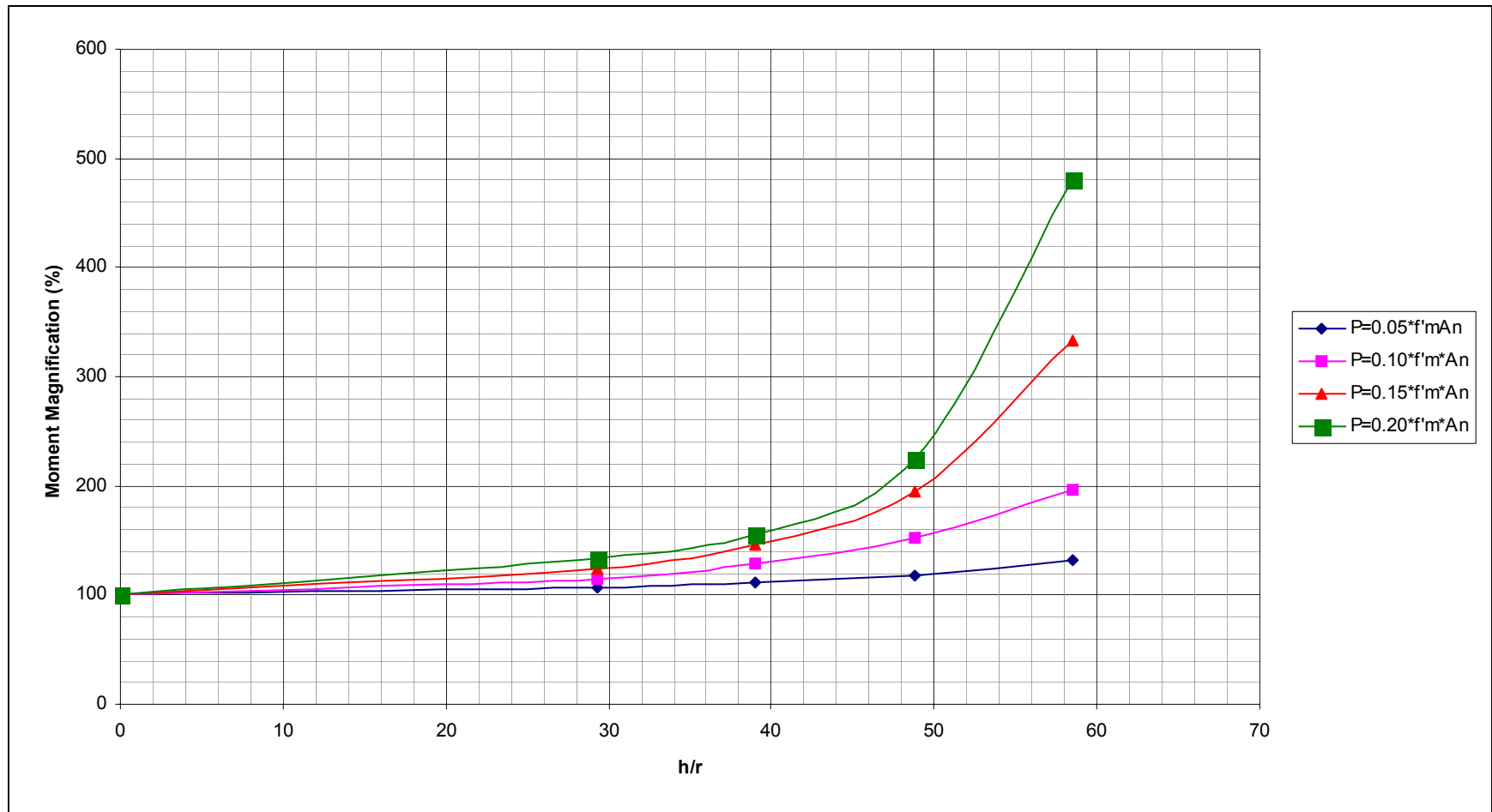


Figure 20. Moment Magnification vs. h/r ratio for 12"CMU, #6@24"

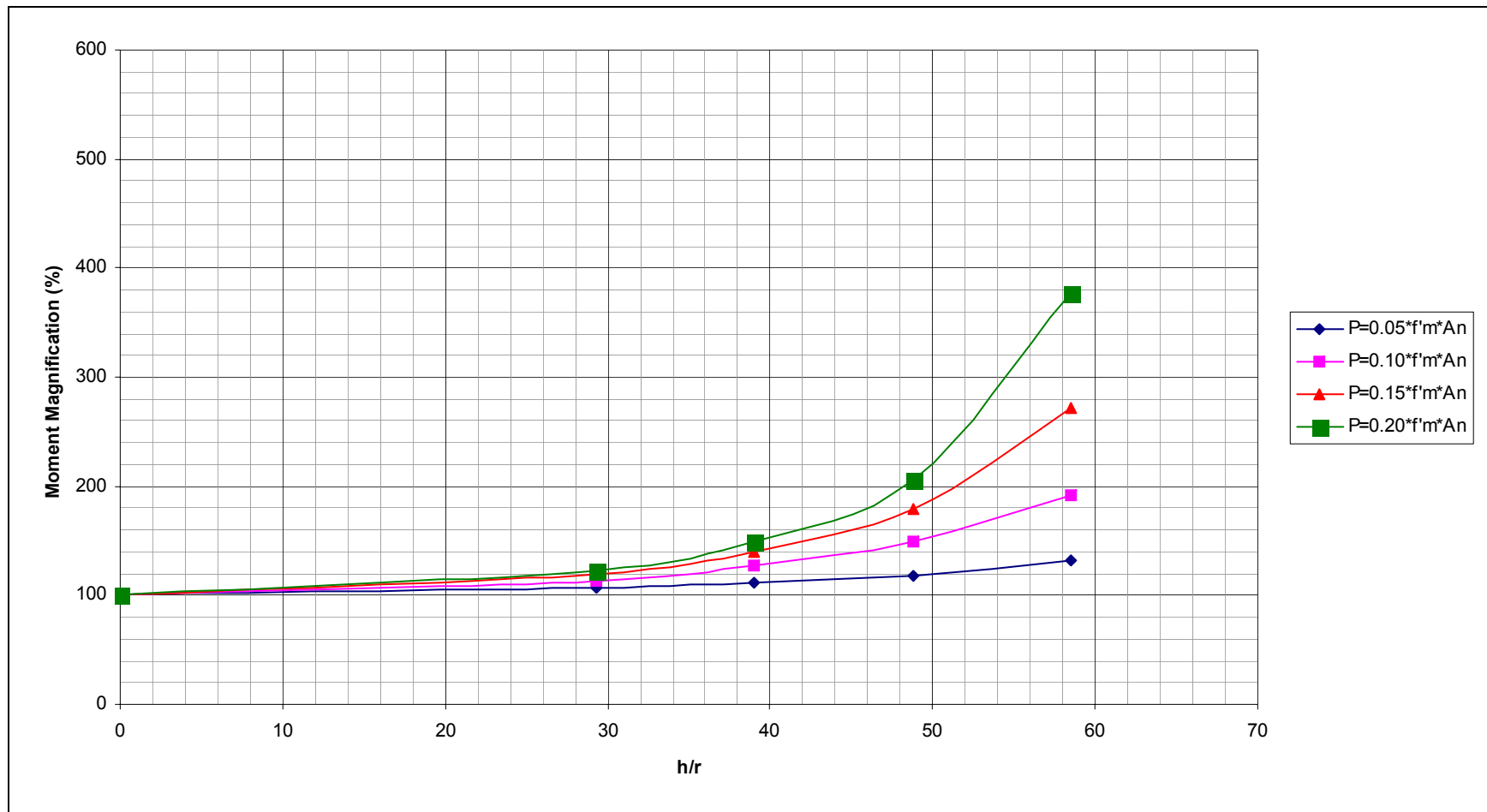


Figure 21. Moment Magnification vs. h/r ratio for 12"CMU, #7@24"

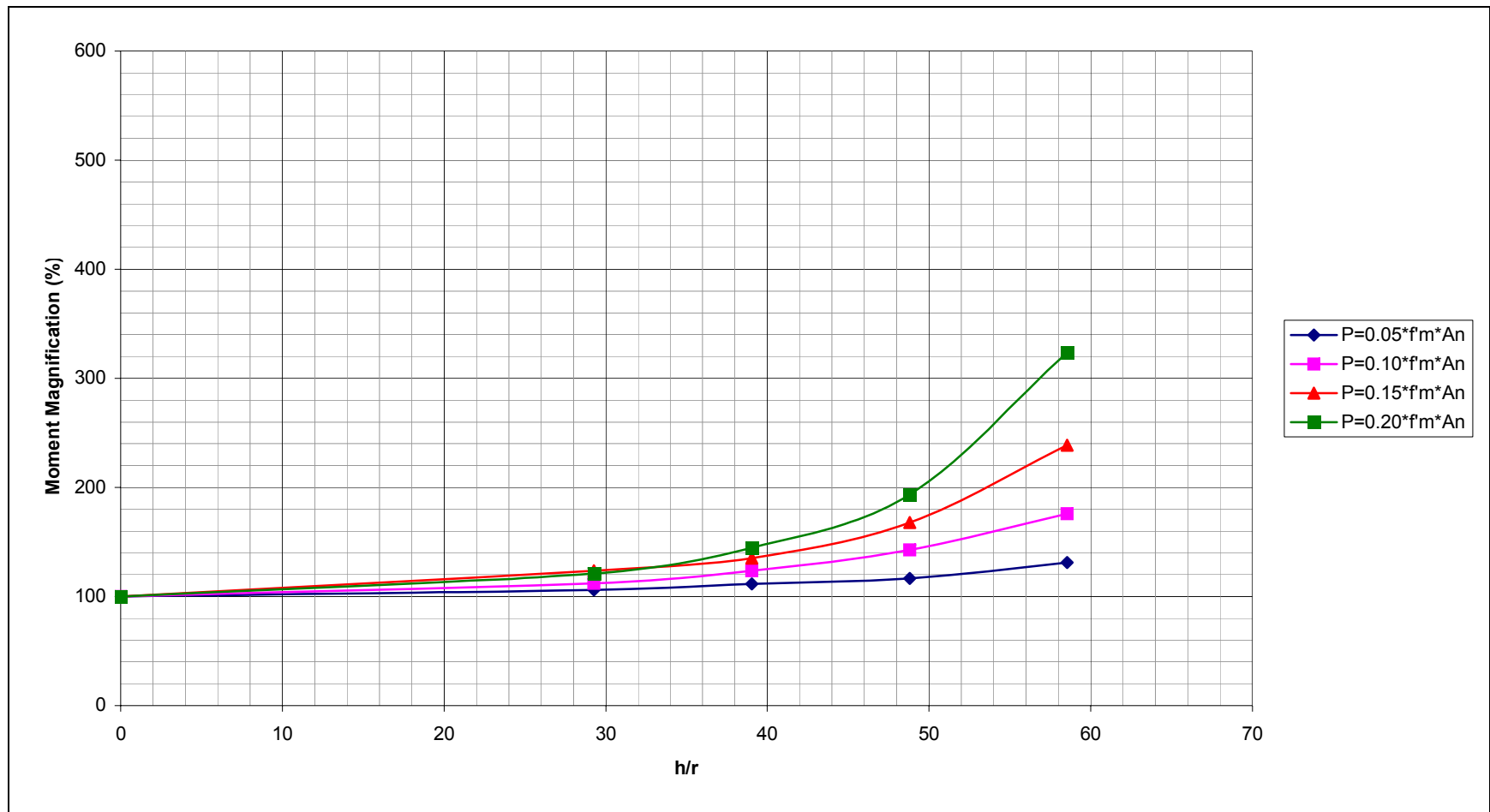


Figure 22. Moment Magnification vs. h/r ratio for 12"CMU, #8@24"

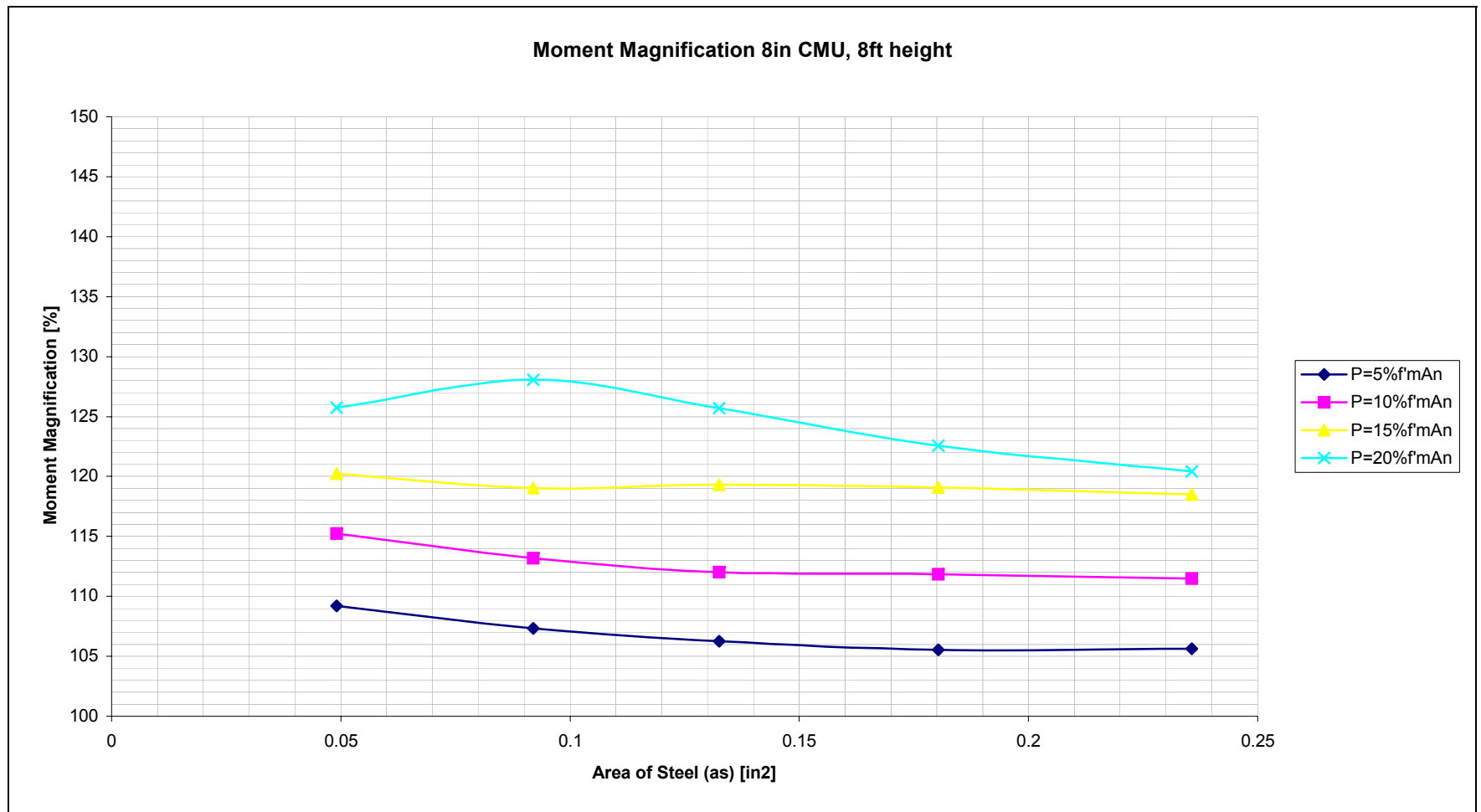


Figure 23. Moment Magnification vs. area of steel for 8"CMU, 8ft tall

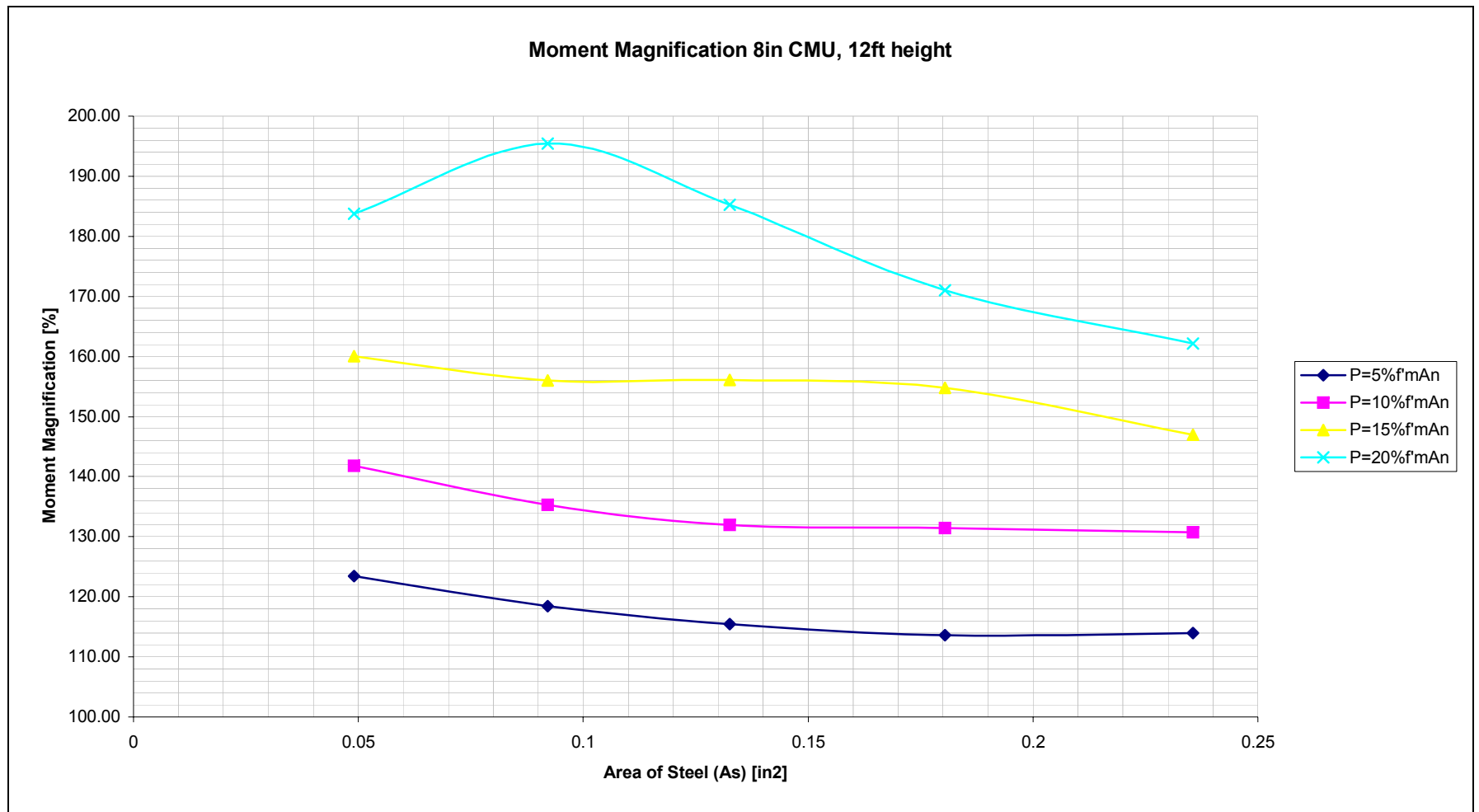


Figure 24. Moment Magnification vs. area of steel for 8"CMU, 12ft tall

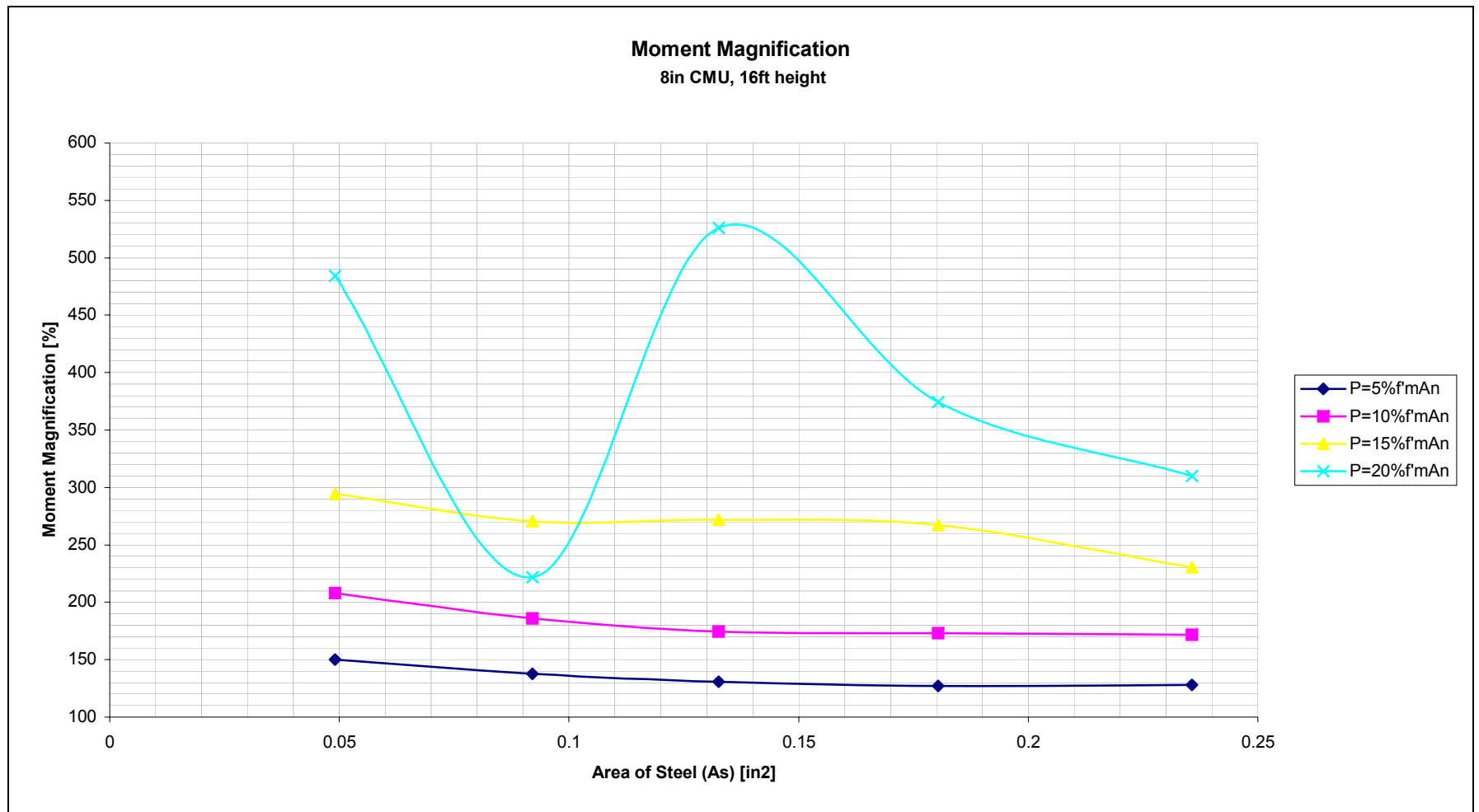


Figure 25. Moment Magnification vs. area of steel for 8"CMU, 16ft tall

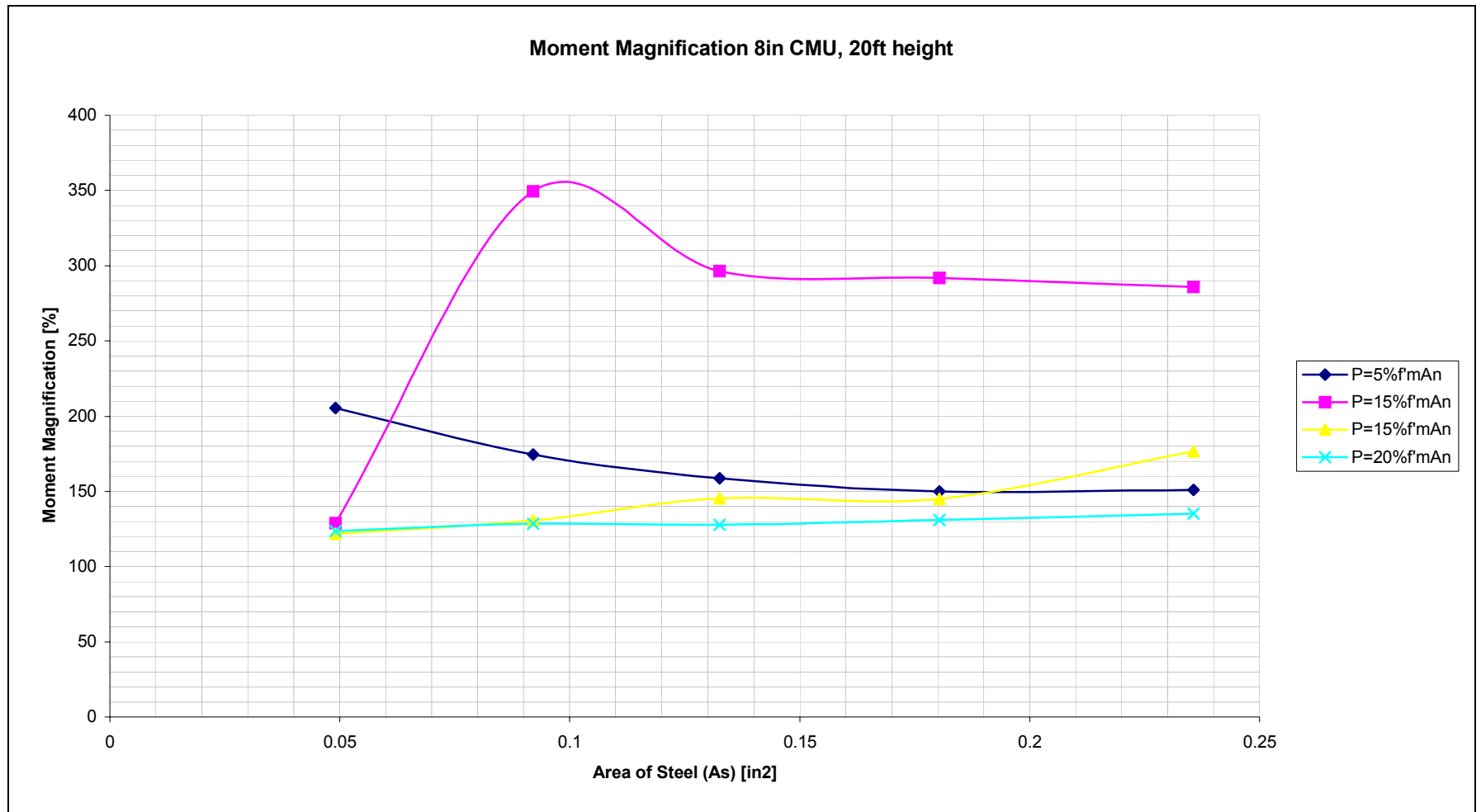


Figure 26. Moment Magnification vs. area of steel for 8"CMU, 20ft tall

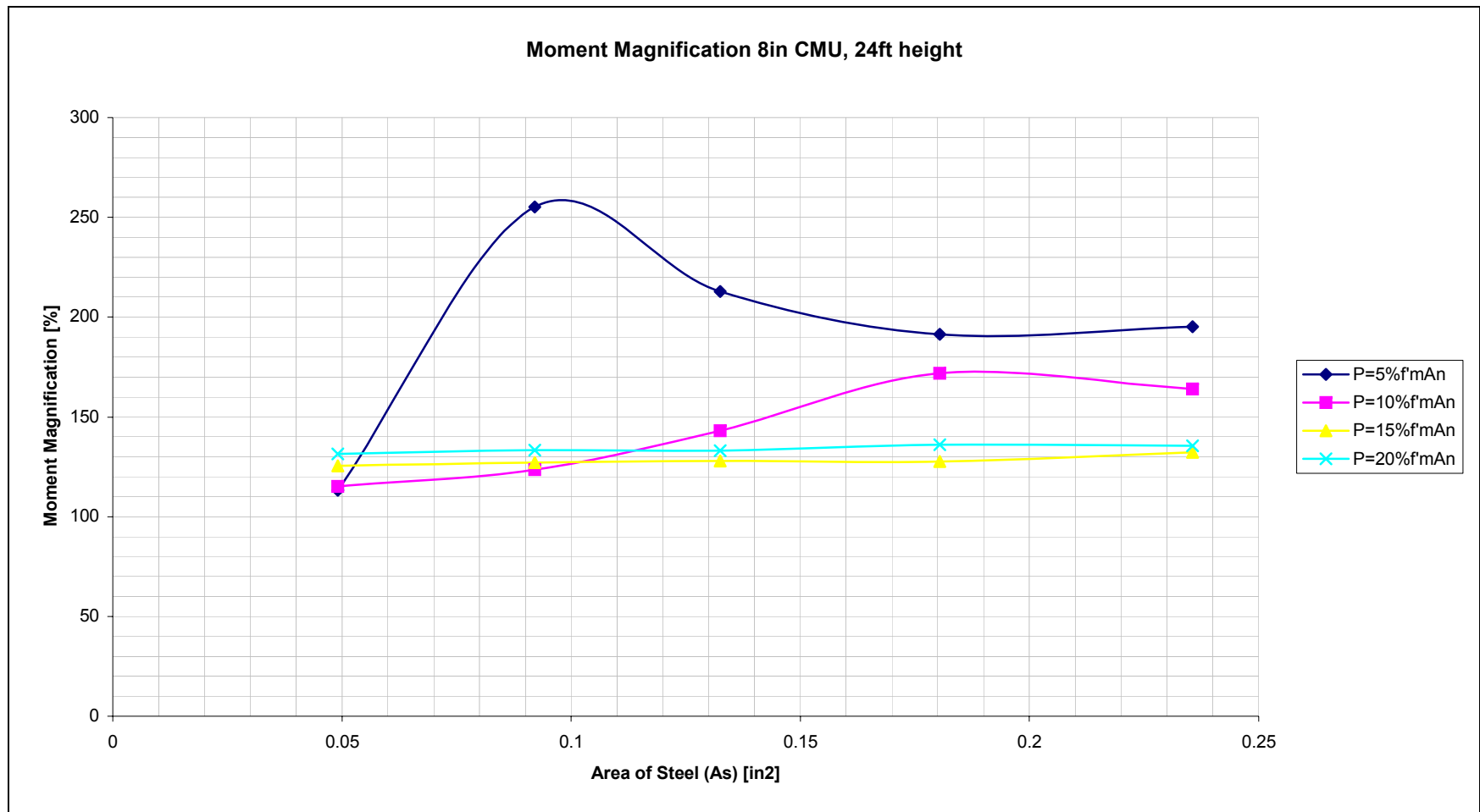


Figure 27. Moment Magnification vs. area of steel for 8"CMU, 24ft tall

Vita

Jarred Wade Cowart was born in Temple, Texas on August 11, 1975. He grew up in Groves, Texas where he attended Port Neches-Groves High School graduating in 1993. After attending Tomball College, he graduated in 1997 from the University of Texas in 1997 with a Bachelor of Arts degree with a major in government.

While at the University of Texas he met his future wife, Jami Erin Meredith who lived in Nashville, Tennessee at the time. After they were married, Jarred attended the University of Tennessee and graduated in 2004 with a Bachelor of Science in civil engineering. In July of 2006 his first son, Jack Fulton, was born.

Jarred is currently pursuing his Master's of Science in structural engineering.